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B. Riemann

Theory of Multifoil Collison Supercompression

V. A. Belokogne

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Editorial

Shock waves and their propagation have been one of the most puzzling phenomena in physics since their prediction by Bernhard Riemann more than 100 years ago. The "state of the art" has not changed much since 1949, when John von Neumann said: "To this day, the only thing of any degree of generality that we possess [on the general hydrodynamical motions in one dimension] is the classical discussion by Riemann, and this very strictly in one dimension and very strictly in the isentropic case."

This issue of the IJFE includes two articles on the subject: the first and only English translation of Riemann's profound and seminal 1859 paper on shock waves. This paper, long misinterpreted and subject to the unfortunate status of a dusty classic, deserves a concentrated examination by all workers in the field. The second article, by Dr. Belokogne of the Soviet Union, contains new and provocative applications of the Riemann's ideas to shock waves in inertial confinement fusion.

Readers of the IJFE will be happy to know that the Fusion Energy Foundation has recently received a grant to reproduce all of Riemann's papers (now at the archives at the University of Göttingen) and will, in the coming issues, be presenting English translations of many of Riemann's previously unpublished mathematical and physical researches.

Authors wishing to submit manuscripts for publication in IJFE *should send two (double-spaced) copies of their work with stats of all figures to Managing Editor,* The International Journal of Fusion Energy, *Fusion Energy Foundation, 888 7th Ave., Suite 2404, New York, NY 10019.*

*On the Propagation of Plane Air Waves of Finite Amplitude**

B. Riemann

Translated from German by U. Parpart and S. Bardwell

Although the differential equations determining the motion of gases have long been established, their integration nonetheless has been carried out only for the case in which the differences in pressure can be viewed as infinitesimal fractions of the entire pressure, and until most recently, it was considered satisfactory to take into account only the first powers of these fractions. Only very recently, Helmholtz in his computation took into account second-order terms, and from this explained the objective generation of combination tones. However, for the case that the initial motion occurs everywhere in the same direction, and that in each plane perpendicular to this direction, pressure and velocity are constant, the exact differential equations can be completely integrated; and even though, for the explanation of phenomena experimentally established so far, the treatment of the equations made up to this point is entirely sufficient, it is nonetheless possible that given the great progress which has been made in the experimental treatment of acoustic questions, the results of this more precise computation may, in the not too distant future, provide certain reference points for experimental research; and this may, apart from the theoretical interest which

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*Translated from the German: B. Riemann, *Gesammelte Mathematische Werke,* ed. H. Weber (Leipzig, 1876), pp. 145–164.

attaches to the treatment of nonlinear partial differential equations, justify the communication of the same.

For the dependency of the pressure on the density, Boyle's Law would have to be assumed if the temperature of the gas could be regarded as constant. The heat exchange, however, can probably be neglected entirely, and, therefore, for this dependency, we must assume the law according to which the pressure of the gas changes with its density when no heat is gained or lost.

According to Boyle's and Gay-Lussac's Law if v is the volume per unit mass, p is the pressure, and T is the absolute temperature, then

$$\log p + \log v = \log T + \text{const.}$$

If we here regard T as a function of p and v and call c the specific heat at constant pressure, and c' the specific heat at constant volume, both with unit mass, then this unit mass, when p and v change by dp and dv, will gain the amount of heat

$$c \frac{\partial T}{\partial v} dv + c' \frac{\partial T}{\partial p} dp$$

or since

$$\frac{\partial \log T}{\partial \log v} = \frac{\partial \log T}{\partial \log p} = 1,$$

$$T(c d \log v + c' d \log p).$$

Thus, when no heat gain occurs, then

$$d \log p = -\frac{c}{c'} d \log v$$

and also if we assume with Poisson that the ratio of the two specific heats $(c/c') = k$ is independent of temperature and pressure, then $\log p = -k \log v + \text{const.}$

According to recent experiments, Regnault, Joule and W. Thomson, these laws are probably very nearly valid for oxygen, nitrogen, and hydrogen, and their mixtures under all realizable conditions. For these gases, Regnault established a very close fit with the law of Boyle and Gay-Lussac and the independence of the specific heat c from temperature and pressure.

For atmospheric air, Regnault found that between $-30°C$ and $+10°C$, $c = 0.2377$; between $+10°C$ and $+100°C$, $c = 0.2379$; between

$+100°C$ and $+215°C$, $c = 0.2376$. Similarly, for pressures from 1 to 10 atm, no noticeable difference in specific heat resulted.

According to experiments by Regnault and Joule, it further appears that for these gases, the assumption adopted by Clausius from Mayer, that a gas expanding at constant temperature will absorb only as much heat as is necessary for the production of the external work, is very nearly correct. When the volume of the gas changes by dv while the temperature remains constant, $d \log p = -d \log v$, the amount of heat absorbed is $T(c - c')d \log v$, and the work performed, is pdv. This hypothesis therefore, when A signifies the mechanical equivalent of heat, yields

$$AT(c - c')d \log v = pdv$$

or $c - c' = (pv/AT)$, thus independent of pressure and temperature.

According to this $k = (c/c')$ is also independent of pressure and temperature. When $c = 0.237733$, A, according to Joule, is 424.55 J, and, for a temperature of 0°C or 272.85°K, pv according to Regnault is 7990.267 J, then $k = 1.4101$. The speed of sound in dry air at 0°C is

$$\sqrt{(7.990.267)(9.8088k)} \text{ m/sec}$$

and with this value of k, is found to be 332.440. The two most complete series of experiments by Moll and van Beek for this calculated separately yield 332.528 m/sec and 331.867 m/sec, jointly yield 332.271 m/sec, and the experiments of Martins and A. Bravais according to their own computation give 332.37 m/sec.

1

To begin with, it is not necessary to make a definite assumption about the dependency of the pressure on the density; we thus assume that for the density, ρ the pressure, is $\varphi(\rho)$, and for the time being we leave the function φ undetermined.

We now assume that orthogonal coordinates x, y, z have been introduced, the x-axis in the direction of the motion—and we let the density be denoted by ρ, the pressure by p, the velocity for coordinate x at time t by u and an element of the plane with coordinates x by ω.

Then the volume of the right cylinder of height dx standing on the element is $\omega\, dx$, the mass contained in it $\omega\rho\, dx$. The change of this mass during the time dt, i.e., the magnitude $\omega\, (\partial\rho/\partial t)\, dt\, dx$ is determined by the mass flowing into it which is found to be equal to $-\omega\, (\partial\rho u/\partial x)\, dx\, dt$.

Its acceleration is $(\partial u/\partial t) + u\, (\partial u/\partial x)$ and the force which propels it in the direction of the positive x-axis is $(-\partial p/\partial x)\omega\, dx = -\varphi'(\rho)(\partial\rho/\partial x)\omega\, dx$, where $\varphi'(\rho)$ denotes the derivative of $\varphi(\rho)$. Thus for ρ and u we have the two differential equations

$$\frac{\partial \rho}{\partial t} = -\frac{\partial \rho u}{\partial x}$$

and

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -\varphi'(\rho)\frac{\partial \rho}{\partial x}$$

or

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\varphi'(\rho)\frac{\partial \log \rho}{\partial x}$$

and

$$\frac{\partial \log \rho}{\partial t} + u\frac{\partial \log \rho}{\partial x} = -\frac{\partial u}{\partial x}.$$

If we multiply the second equation by $\pm\sqrt{\varphi'(\rho)}$ add it to the first, and for purposes of abbreviation let

$$\int \sqrt{\varphi'(\rho)}\, d\log\rho = f(\rho) \tag{1}$$

and

$$f(\rho) + u = 2r, \qquad f(\rho) - u = 2s \tag{2}$$

then these equations attain the simpler form

$$\frac{\partial r}{\partial t} = -(u + \sqrt{\varphi'(\rho)})\frac{\partial r}{\partial x},$$

$$\frac{\partial s}{\partial t} = -(u - \sqrt{\varphi'(\rho)})\frac{\partial s}{\partial x}, \tag{3}$$

where u and ρ are functions of r and s determined by Eqs. (2). From these follow

$$dr = \frac{\partial r}{\partial x}\,(dx - (u + \sqrt{\varphi'(\rho)})\,dt) \tag{4}$$

$$ds = \frac{\partial s}{\partial x}\,(dx - (u - \sqrt{\varphi'(\rho)})\,dt) \tag{5}$$

Under the assumption which in reality always obtains that $\varphi'(\rho)$ is positive, these equations express the fact that r remains constant when x changes with t in such a way that $dx = (u + \sqrt{\varphi'(\rho)})\,dt$, and that s remains constant when x changes with t in such a way that $dx = (u - \sqrt{\varphi'(\rho)}\,dt$. Thus a specific value of r or of $f(\rho) + u$ advances to larger values of x with the velocity $\sqrt{\varphi'(\rho)} + u$, while a specific value of s or of $f(\rho) - u$ decreases to smaller values of x with the velocity $\sqrt{\varphi'(\rho)} - u$.

Thus a specific value of r will sooner or later coincide with every previously realized value of s and the velocity of its advance will at each moment depend on that value of s with which it coincides.

2

The analysis, now first of all offers the means to answer the question where and when a specific value of r, call it r', meets a specific value of s, s', i.e., to determine x and t as functions of r and s. Indeed, if in the Eqs. (3) of the previous section, we introduce r and s as independent variables, then these equations are transformed into linear differential equations for x and t and can be integrated therefore, according to known methods. In order to effect the reduction of the differential equations to linear ones, it is most useful to put Eqs. (4) and (5) of the previous section into the form

$$dr = \frac{\partial r}{\partial x}\left\{d(x - (u + \sqrt{\varphi'(\rho)})t) + \left[dr\left(\frac{d\log\sqrt{\varphi'(\rho)}}{d\log\rho} + 1\right) + ds\left(\frac{d\log\sqrt{\varphi'(\rho)}}{d\log\rho} - 1\right)\right]t\right\} \tag{1}$$

$$ds = \frac{\partial s}{\partial x}\left\{d(x - (u - \sqrt{\varphi'(\rho)})t) - \left[ds\left(\frac{d\log\sqrt{\varphi'(\rho)}}{d\log\rho} + 1\right) + dr\left(\frac{d\log\sqrt{\varphi'(\rho)}}{d\log\rho} - 1\right)\right]t\right\}. \tag{2}$$

One then obtains two linear differential equations for x and t, when s and r are regarded as independent variables

$$\frac{\partial(x - (u + \sqrt{\varphi'(\rho)})t)}{\partial s} = -t\left(\frac{d \log \sqrt{\varphi'(\rho)}}{d \log \rho} - 1\right)$$

$$\frac{\partial(x - (u + \sqrt{\varphi'(\rho)})t)}{\partial r} = t\left(\frac{d \log \sqrt{\varphi'(\rho)}}{d \log \rho} - 1\right).$$

As a result of these

$$(x - (u + \sqrt{\varphi'(\rho)})t)\, dr - (x - (u - \sqrt{\varphi'(\rho)})t)\, ds \tag{3}$$

is a complete differential whose integral, w, satisfies the equation:

$$\frac{\partial^2 w}{\partial r\, \partial s} = -t\left(\frac{d \log \sqrt{\varphi'(\rho)}}{d \log \rho} - 1\right) = m\left(\frac{\partial w}{\partial r} + \frac{\partial w}{\partial s}\right)$$

where

$$m = \frac{1}{2\sqrt{\varphi'(\rho)}}\left(\frac{d \log \sqrt{\varphi'(\rho)}}{d \log \rho} - 1\right)$$

i.e., a function of $r \pm s$. If we let $f(\rho) = r + s = \sigma$, then

$$\sqrt{\varphi'(\rho)} = \frac{d\sigma}{d \log \rho},$$

and consequently

$$m = -\frac{1}{2}\frac{d \log \dfrac{d\rho}{d\sigma}}{d\sigma}.$$

With the Poisson assumption $\varphi(r) = aa\rho^k$, we get

$$f(\rho) = \frac{2a\sqrt{k}}{k-1}\rho^{(k-1)/2} + \text{const.},$$

and if for the arbitrary constant we choose the value zero,

$$\sqrt{\varphi'(\rho)} + u = \frac{k+1}{2} r + \frac{k-3}{2} s$$

$$\sqrt{\varphi'(\rho)} - u = \frac{k-3}{2} r + \frac{k+1}{2} s$$

$$m = \left(\frac{1}{2} - \frac{1}{k-1}\right)\frac{1}{\sigma} = \frac{k-3}{2(k-1)(r+s)}\,.$$

Under the assumption of Boyle's law $\varphi(\rho) = aa\rho$ we obtain values

$$f(\rho) = a \log \rho$$

$$\sqrt{\varphi'(\rho)} + u = r - s + a$$

$$\sqrt{\varphi'(\rho)} - u = s - r + a$$

$$m = -\frac{1}{2a}$$

which follow from the above if one reduces $f(\rho)$ by the constant $(2a\sqrt{k})/(k-1)$ i.e., r and s by $(a\sqrt{k})/(k-1)$ and then sets $k = 1$.

However, the introduction of r and s as independent variable magnitudes is possible only if the determinant of these functions of x and t which is equal to $2\sqrt{\varphi'(\rho)}(\partial r/\partial x)\,(\partial s/\partial x)$ does not vanish, i.e., only if $(\partial r/\partial x)\,(\partial s/\partial x)$ are both different from zero.

If $(\partial r/\partial x) = 0$, then from Eq. (1) it follows that $dr = 0$ and from Eq. (2) that $x - (u - \sqrt{\varphi'(\rho)})t$ is a function of s. Consequently, then, too, Eq. (3) is a complete differential and w becomes a function only of s.

For similar reasons, if $(\partial s/\partial x) = 0$ and s is constant also with respect to t, then $x - (u + \sqrt{\varphi'(\rho)})t$ and w become functions of r.

Finally, when $\partial r/\partial x$ and $\partial s/\partial x$ are both equal to zero, then in consequence of the differential equations r, s and w become constants.

3

In order to solve the problem, we must now first determine w as a function of r and s in such a way that it satisfies the differential equation

$$\frac{\partial^2 w}{\partial r\,\partial s} - m\left(\frac{\partial w}{\partial r} + \frac{\partial w}{\partial s}\right) = 0 \tag{1}$$

with the initial conditions by means of which it is determined up to a constant which can be added to it arbitrarily.

Where and when a specific value of r coincides with a specific value of s is then given by

$$(x - (u + \sqrt{\varphi'(\rho)})t)dr - (x - (u - \sqrt{\varphi'(\rho)})t)ds = dw; \tag{2}$$

and hereafter one finally finds u and ρ as functions of x and t through utilization of the equations

$$f(\rho) + u = 2r$$

$$f(\rho) - u = 2s. \tag{3}$$

Indeed, unless over a finite segment, dr or ds is zero, and hence r or s is constant, Eq. (2) yields the equations

$$x - (u + \sqrt{\varphi'(\rho)})t = \frac{\partial w}{\partial r}, \tag{4}$$

$$x - (u - \sqrt{\varphi'(\rho)})t = -\frac{\partial w}{\partial s}, \tag{5}$$

through whose connections with Eq. (3) one obtains u and ρ expressed in terms of x and t.

But if r initially has the constant value r' over a finite segment, then in this region, where $r = r'$, the value of $x - (u + \sqrt{\varphi'(\rho)})t$ can then not be determined from Eq. (2) since $dr = 0$; and, indeed, the question of where and when this value r' will meet a specific value of s then does not admit of a definite answer. Equation (4) then holds only at the boundaries of this region, and determines between what values of x at specific time the constant value of r' occurs, or, again, during what period of time at a specific point r this value obtains. Between these boundaries, u and ρ are determined as functions in similar fashion when s possesses the value s' in a finite region in which r is variable, as well as when both r and s are constant. In the latter case, between certain boundaries determined by Eqs. (4) and (5) they assume constant values determined by Eq. (3).

4

Before we undertake the integration of Eq. (1) of the previous section, it would be useful first to present a few considerations which do not presuppose a knowledge of the integration itself. Concerning the funcwith the initial conditions by means of which it is determined up to a constant which can be added to it arbitrarily.

Where and when a specific value of r coincides with a specific value of s is then given by

$$(x - (u + \sqrt{\varphi'(\rho)})t)dr - (x - (u - \sqrt{\varphi'(\rho)})t)ds = dw; \tag{2}$$

and hereafter one finally finds u and ρ as functions of x and t through utilization of the equations

$$\begin{aligned} f(\rho) + u &= 2r \\ f(\rho) - u &= 2s. \end{aligned} \tag{3}$$

Indeed, unless over a finite segment, dr or ds is zero, and hence r or s is constant, Eq. (2) yields the equations

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But if r initially has the constant value r' over a finite segment, then in this region, where $r = r'$, the value of $x - (u + \sqrt{\varphi'(\rho)})t$ can then not be determined from Eq. (2) since $dr = 0$; and, indeed, the question of where and when this value r' will meet a specific value of s then does not admit of a definite answer. Equation (4) then holds only at the boundaries of this region, and determines between what values of x at specific time the constant value of r' occurs, or, again, during what period of time at a specific point r this value obtains. Between these boundaries, u and ρ are determined as functions in similar fashion when s possesses the value s' in a finite region in which r is variable, as well as when both r and s are constant. In the latter case, between certain boundaries determined by Eqs. (4) and (5) they assume constant values determined by Eq. (3).

4

Before we undertake the integration of Eq. (1) of the previous section, it would be useful first to present a few considerations which do not presuppose a knowledge of the integration itself. Concerning the function $\varphi(\rho)$ the only necessary assumption, (which in reality will certainly always be the case) is that with growing ρ its derivative does not decrease. We also note here a point that will be applied on several occasions in the following section, that under these circumstances:

$$\frac{\varphi(\rho_1) - \varphi(\rho_2)}{\rho_1 - \rho_2} = \int_0^1 \varphi'(\alpha\rho_1 + (1 - \alpha)\rho_2)\, d\alpha,$$

when only one of the quantities ρ_1 and ρ_2 change, the equation will either remain constant or increase or decrease along with these quantities respectively. Simultaneously it follows from this, that the value of this expression will always be between $\varphi'(\rho_1)$ and $\varphi'(\rho_2)$.

Let us first consider the case where the initial disturbance of the equilibrium is limited to a finite region determined by the inequality $a < x < b$ so that outside it u and ρ and consequently r and s are constant; let the values of these quantities for $x < a$ be denoted by use of the subscript 1, for $x > b$ by the subscript 2. According to Section 1, the area in which r is variable slowly moves foward and in particular its rear boundary moves with velocity $\sqrt{\varphi'(\rho_1)} + u$, while the forward boundary of the area in which s is variable, goes backwards with the velocity $\sqrt{\varphi'(\rho_2)} - u_2$. After a period of time

$$\frac{b - a}{\sqrt{\varphi'(\rho_1)} + \sqrt{\varphi'(\rho_2)} + u_1 + u_2}.$$

The two regions move apart from each other, and between them, a gap forms in which $s = s_2$ and $r = r_1$, and consequently, the gas particles are in equilibrium again. Hence, from the initially disturbed place, two waves originate propagating in opposite directions. In the advancing one, we have $s = s_2$; therefore, with the specific value of the density ρ there is always connected the velocity $u = f(\rho) - 2s_2$ and both values advance with the constant velocity

$$\sqrt{\varphi'(\rho)} + u = \sqrt{\varphi'(\rho)} + f(\rho) - 2s_2.$$

In the receding wave on the other hand, the velocity $-f(\rho) + 2r_1$ is connected with the density ρ and these two values recede with the velocity $\sqrt{\varphi'(\rho)} + f(\rho) - 2r_1$. For greater densities, the propagation velocity is greater since both $\sqrt{\varphi'(\rho)}$ and $f(\rho)$ increase along with ρ.

If we think of ρ as the ordinate of a curve for the abscissa x, then this point moves along parallel to the abscissa with constant velocity which is larger for larger values of the ordinate. One easily realizes that with this law points of greater ordinate would ultimately overtake precursor points with smaller ordinates so that to one value of x there would correspond more than one value of ρ. Now, since in reality this cannot take place, a circumstance must arise as a result of which this law becomes invalid. And, indeed, the derivation of the differential equations was based on the assumption that u and ρ are continuous functions of x and have finite derivatives; this assumption, however, ceases to be fulfilled as soon as at any point the density curve becomes perpendicular to the abscissa, and from this point on a discontinuity arises in this curve so that a larger value of ρ immediately follows upon a smaller one; a case which will be discussed in the next section.

The compression waves, i.e., those parts of the wave in which the density decreases in the direction of propagation, accordingly will become more and more narrow as they progress, and will ultimately change into compression shocks; the width of the rarefaction waves, however, grows continuously proportional with time.

It is easy to show, at least under the assumption of Poisson's (or Boyle's) law, that, even when the initial disturbance of the equilibrium is not limited to a finite region, compression shocks must always (except in very special cases) form in the course of the motion. The velocity with which a value of r advances is, under this assumption

$$\frac{k+1}{2}r + \frac{k-3}{2}s;$$

greater values thus will on the average move with greater velocity and a larger value r' must ultimately catch up with a smaller precursor value r'', unless the value of s coincident with r'' is smaller by $(r' - r'')[(1+k)/(3-k)]$ than that value of s simultaneously coincident with r'. In this case for a positive infinite x, s would become negative infinite, and consequently, for $x = +\infty$, the velocity would become $u = +\infty$ (or instead, in the case of Boyle's law, the density would become infinitely small). Excepting special cases, therefore, a value of r greater by a finite quantity must always immediately follow a smaller one; consequently, because $\partial r/\partial x$ becomes infinite, the differential equations will lose their validity and advancing compression shocks must arise. Similarly, we will almost always get receding compression shocks as a result of the fact that $\partial s/\partial x$ becomes infinite.

For the determination of the time and place for which $\partial r/\partial x$ or $\partial s/\partial x$ become infinite, and sudden compressions originate, we obtain from Eqs. (1) and (2) of Section 2, introducing there the function w

$$\frac{\partial r}{\partial x}\left(\frac{\partial^2 w}{\partial r^2} + \left(\frac{d \log \sqrt{\varphi'(\rho)}}{d \log \rho} + 1\right)t\right) = 1,$$

$$\frac{\partial s}{\partial x}\left(-\frac{\partial^2 w}{\partial s^2}-\left(\frac{d\log\sqrt{\varphi'(\rho)}}{d\log\rho}+1\right)t\right)=1.$$

5

We must now, since sudden compressions almost always take place (even if density and velocity initially vary continuously everywhere), seek to determine the laws for the propagation of compression shocks.

We assume that at time t for $x=\xi$, a jumplike change in u and ρ take place, and we denote the values of these variables and of the variables dependent on them for $x=\xi-0$ by attaching the subscript 1, and for $x=\xi+0$ by the subscript 2; let the relative velocities, with which the gas moves towards the point of discontinuity, $u_1-(d\xi/dt)$, $u_2-(d\xi/dt)$, be denoted by v_1 and v_2. The mass, which passes through a surface element ω at $x=\xi$ in a time dt, in a positive direction, is equal to $v_1\rho_1\omega\,dt=v_2\rho_2\omega\,dt$. The force impressed on it is $(\varphi(\rho_1)-\varphi(\rho_2))\omega\,dt$ and the increase in velocity brought about by this v_2-v_1; therefore we have:

$$(\varphi(\rho_1)-\varphi(\rho_2))\omega\,dt=(v_2-v_1)v_1\rho_1\omega\,dt$$

and

$$v_1\rho_1=v_2\rho_2,$$

from which it follows

$$v_1=\mp\sqrt{\frac{\rho_2}{\rho_1}\frac{\varphi(\rho_1)-\varphi(\rho_2)}{\rho_1-\rho_2}},$$

thus

$$\frac{d\xi}{dt}=u_1\pm\sqrt{\frac{\rho_2}{\rho_1}\frac{\varphi(\rho_1)-\varphi(\rho_2)}{\rho_1-\rho_2}}$$

$$=u_2\pm\sqrt{\frac{\rho_2}{\rho_1}\frac{\varphi(\rho_1)-\varphi(\rho_2)}{\rho_1-\rho_2}}. \qquad (1)$$

For a compression shock $\rho_2-\rho_1$ must have the same sign as v_1 and v_2, and in particular the negative sign for an advancing shock, and a positive sign for the receding one. In the first case, the upper sign holds and ρ_1 is larger than ρ_2; therefore, given the assumption at the beginning of the previous section concerning the function $\varphi(\rho)$

If we denote the values taken on by u and ρ behind or in between the compression shocks in the first instant of their progression with a prime, then in the first case, $\rho' > \rho_1$ and $> \rho_2$ and we have

$$
\begin{aligned}
u_1 - u' &= \sqrt{\frac{(\rho' - \rho_1)(\varphi(\rho') - \varphi(\rho_1))}{\rho'\rho_1}} \\
u' - u_2 &= \sqrt{\frac{(\rho' - \rho_2)(\varphi(\rho') - \varphi(\rho_2))}{\rho'\rho_2}}
\end{aligned}
\tag{1}
$$

$$
u_1 - u_2 = \sqrt{\frac{(\rho' - \rho_1)(\varphi(\rho') - \varphi(\rho_1))}{\rho'\rho_1}} + \sqrt{\frac{(\rho' - \rho_2)(\varphi(\rho') - \varphi(\rho_2))}{\rho'\rho_2}}. \tag{2}
$$

Thus, since both terms of the right hand side of Eq. (2) grow simultaneously with ρ', $u_1 - u_2$ must be positive and

$$
(u_1 - u_2)^2 > \frac{(\rho_1 - \rho_2)(\varphi(\rho_1) - \varphi(\rho_2))}{\rho_1\rho_2};
$$

and conversely when these conditions are satisfied there always exists one and only one pair of values of u' and ρ' satisfying Eqs. (1).

In order for the last case to occur, and thus for the motion to be determinable according to the differential equations it is necessary and sufficient that $r_1 \leq r_2$ and $s_1 \geq s_2$ thus that $u_1 - u_2$ be negative and $(u_1 - u_2)^2 \geq (f(\rho_1) - f(\rho_2))^2$. Under these conditions the difference between r_1 and r_2, and s_1 and s_2 grows as they progress, since the preceding value progresses with greater velocity, and thus the discontinuity disappears.

When neither the former nor the latter conditions are satisfied then a single compression shock satisfies the initial values, either an advancing or receding one, depending on whether ρ_1 is greater or smaller than ρ_2.

Indeed, when $\rho_1 > \rho_2$,

$$
2(r_1 - r_2) \text{ or } f(\rho_1) - f(\rho_2) + u_1 - u_2
$$

is positive, since $(u_1 - u_2)^2 < (f(\rho_1) - f(\rho_2))^2$ and simultaneously

$$
\leq f(\rho_1) - f(\rho_2) + \sqrt{\frac{(\rho_1 - \rho_2)(\varphi(\rho_1) - \varphi(\rho_2))}{\rho_1\rho_2}}
$$

since

$$(u_1 - u_2)^2 \leq \frac{(\rho_1 - \rho_2)(\varphi(\rho_1) - \varphi(\rho_2))}{\rho_1 \rho_2};$$

thus for the density ρ' behind the compression shock a value satisfying condition (3) of the previous section can be found and it is $\leq \rho_1$. Consequently, since $s' = f(\rho') - r_1$, $s_1 = f(\rho_1) - r_1$, we also have $s' \leq s_1$ so that the motion behind the compression shock can occur in accordance with the differential equations.

Clearly, the other case when $\rho_1 < \rho_2$ is not essentially different from this case.

7

In order to explicate things up until now, by means of a simple example, in which the motion can be determined on the basis of the tools developed so far, we shall assume that pressure and density depend on each other in accordance with Boyle's law and pressure and velocity have a jump at $x = 0$ but are constant on both sides of this point. According to the above, we must distinguish four cases:

I.

If $u_1 - u_2 > 0$ and thus the two masses of gas move towards each other and

$$\left(\frac{u_1 - u_2}{a}\right)^2 > \frac{(\rho_1 - \rho_2)^2}{\rho_1 \rho_2}$$

then two compression shocks running in opposite directions are created. According to Section 6, Eq. (1), if $\sqrt[4]{\rho_1/\rho_2}$ is denoted by α and the positive root of the equation

$$\frac{u_1 - u_2}{a\left(\alpha + \dfrac{1}{\alpha}\right)} = \theta - \frac{1}{\theta}$$

by θ, the density between the compression shocks is $\rho' = \theta\theta\sqrt{\rho_1 \rho_2}$, and according to Section 5 eq. (1), we have for the advancing compression shock

$$\frac{d\xi}{dt} = u_2 + a\alpha\theta = u' + \frac{a}{\alpha\theta},$$

III.

If neither of these cases obtain and $\rho_1 > \rho_2$, then a rarefaction wave running backwards and a compression shock running forwards arise. For the latter we find from Section 5 [Eq. (3)], if θ denotes the root of the equation

$$\frac{2(r_1 - r_2)}{a} = 2 \log \theta + \theta - \frac{1}{\theta},$$

that $\rho' = \theta\theta\rho_2$, and from Section 5 [Eq. (1)]:

$$\frac{d\xi}{dt} = u_2 + a\theta = u' + \frac{a}{\theta}.$$

After the passage of time t, we therefore have in front of the compression shock, i.e., when $x > (u_2 + a\theta)t$, $u = u_2$, $\rho = \rho_2$, while behind the compression shock we have $r = r_1$ and furthermore, if

$$(u_1 - a)t < x < (u' - a)t, u = a + \frac{x}{t};$$

for smaller x, $u = u_1$ and for larger x, $u = u'$.

IV.

If, finally, the first two cases do not obtain and $\rho_1 < \rho_2$, then the development is just like in III, only in the opposite direction.

8

In order to solve our problem in general terms, according to Section 3 the function w must be determined in such a way that it satisfies the differential equation

$$\frac{\partial^2 w}{\partial r\, \partial s} - m\left(\frac{\partial w}{\partial r} + \frac{\partial w}{\partial s}\right) = 0 \tag{1}$$

and the initial conditions.

If we exclude the occurrence of discontinuities, then clearly according to Section 1, place and time or the values of x and t, for which a specific value r' of r coincides with a specific value s' of s, are completely determined, when the initial values of r and s for the segment between the two values r' and s' are given and if everywhere in the domain (S), which for each value of t, includes those values of x which lie between two points where $r = r'$ and $s = s'$, the differential Eqs. (3) of Section 1 are satisfied. Thus, the value of w, too, is completely determined for $r = r'$ and $s = s'$, if everywhere in the domain (S), w satisfies differential Eq. (1) and if for the initial values of r and s, the values of and $\partial w/\partial r$ and $\partial w/\partial s$ and hence, apart from an additive constant, also of w are given, and provided that this constant has been arbitrarily chosen. For these conditions are equivalent to those above. It further follows from Section 3, that while $\partial w/\partial r$ on both sides of a value r'' of r, if this value occurs within a finite segment, takes on different values, it nonetheless changes everywhere continuously with s; similarly $\partial w/\partial s$ changes with r, while the function w itself changes everywhere continuously with both r and s.

After these preparations, we can now approach the solution of our problem, i.e., the determination of the value of w, for two arbitrary values, r' and s', of r and s.

For purposes of visualization, let us now think of x and t as the abscissa and ordinate of a point in a plane, and assume that in this plane curves of constant r and constant s have been drawn. Of these curves let the former be denoted by (r) and the latter by (s), and on them let the direction of increasing t be regarded as positive. The domain (S), is then represented by a piece of the plane which is delimited by the curve (r'), curve (s'), and a piece of the abscissa between them, and we are concerned with determining the value of w at the point of intersection for given values of both for the former and the latter lines. We wish to generalize the problem somewhat and suppose that the domain (S), is delimited not by this latter line, but by any curve c which cuts neither of the curves (r) and (s) more than once, and that for this curve the values $\partial w/\partial r$ and $\partial w/\partial s$ belonging to the pair r and s are given. As will be apparent from the solution to the problem, these values of $\partial w/\partial r$ and $\partial w/\partial s$ will only be subject to the condition that they remain continuous with changes in position on the curve, even when the curve c cuts one of the curves (r) or (s) more than once. By the way, these values can be assumed to be arbitrary, as long as they do not become independent of each other.

In order to determine which functions should satisfy the linear partial differential equations and the linear boundary conditions, one can use a completely similar procedure, as when one adds to the solution of a system of linear equations all the equations, multiplied by undetermined factors, these factors then being determined in such a way that all but one of the unknown quantities drops out of the sum.

Imagine that a piece (S) of the plane is cut into infinitesimal parallelograms by the curves (r) and (s), and signify by δr and δs the changes that the quantities r and s undergo when the curve element that forms the sides of these parallelograms is traversed in the positive direction; further, signify by v an arbitrary function of r and s that is everywhere continuous and has a continuous derivative. As a result of Eq. (1), one then has

$$0 = \int v \left(\frac{\partial^2 w}{\partial r\, \partial s} - m \left(\frac{\partial w}{\partial r} + \frac{\partial w}{\partial s} \right) \right) \delta r\, \delta s \tag{2}$$

extending over the entire domain (S). Now one must rearrange the order of unknowns on the right-hand side of the equation, i.e., the integral must here be transformed by partial integration, so that, aside from the known quantities, it contains only the desired function, not its derivatives. As a result of this operation the integral is transformed, for the present, into the following integral extending over (S)

$$\int w \left(\frac{\partial^2 v}{\partial r\, \partial s} + \frac{\partial m v}{\partial r} + \frac{\partial m v}{\partial s} \right) \delta r\, \delta s$$

and a simple integral which, since dw/dr varies continuously with s, as does dw/ds with r, and as does w with both variables, extends only over the boundary of (S). By dr and ds, I mean the changes in r and s in an element of the boundary, when the boundary is traversed in the direction which lies with respect to the direction toward the interior just as the positive direction along the curve (r) lies with respect to the positive direction along the curve (s); then this boundary integral is

$$= -\int \left(v \left(\frac{\partial w}{\partial s} - m w \right) ds + w \left(\frac{\partial v}{\partial r} + m v \right) dr \right).$$

The integral along the entire boundary of S is equal to the sum of the integrals along the curves c, (s'), (r'), which form this boundary. Thus, when their points of intersection are called (c, r'), (c, s'), (r', s')

$$= \int_{c,r'}^{c,s'} + \int_{c,s'}^{r,s'} + \int_{s',r'}^{c,r'}$$

In these three components, the first contains, aside from the function v only known quantities; the second, since in it $ds = 0$, contains only the unknown function w itself, not its derivatives; the third component can be transformed by partial integration

$$(vw)_{r,s'} - (vw)_{c,r'} + \int_{s',r'}^{c,r'} w \left(\frac{\partial v}{\partial s} + m v \right) ds$$

so that in it likewise only the desired function w itself occurs.

By assuming Poisson's law, according to which

$$m = \left(\frac{1}{2} - \frac{1}{k-1}\right)\frac{1}{\sigma},$$

one can express ψ_1 and ψ_2 by definite integrals, so that one obtains a triple integral for v, which by this reduction yields

$$v = \left(\frac{r' + s'}{r + s}\right)^{(1/2)-[1/(k-1)]} \cdot F\left(\frac{3}{2} - \frac{1}{k-1}, \frac{1}{k-1} - \frac{1}{2}, 1, -\frac{(r-r')(s-s')}{(r+s)(r'+s')}\right).$$

One can now easily show the correctness of this expression, by showing that it actually satisfies the conditions (3) of the previous section.

One sets $v = e^{-\int_{\sigma'}^{\sigma} m d\sigma} y$, then these conditions are transformed in terms of y

$$\frac{\partial^2 y}{\partial r\, \partial s} + \left(\frac{dm}{d\sigma} - mm\right) y = 0,$$

and $y = 1$ both for $r = r'$ and for $s = s'$. According to Poisson's assumption, one can satisfy these conditions, as long as one assumes that y is a function of $z = [(r - r')(s - s')/(r + s)(r' + s')]$. Then, $m = (\lambda/\sigma)$, when one symbolizes $1/2 - 1/(k - 1)$ by λ. Thus $[(dm/d\sigma) - mm] = (-\lambda + \lambda^2/\sigma^2)$ and

$$\frac{\partial^2 y}{\partial s\, \partial r} = \frac{1}{\sigma^2}\left(\frac{d^2 y}{d \log z^2}\left(1 - \frac{1}{z}\right) + \frac{dy}{d \log z}\right).$$

Consequently, $v = (\sigma'/\sigma)^\lambda y$ and y is a solution to the differential equation

$$(1 - z)\frac{d^2 y}{d \log z^2} - z\frac{dy}{d \log z} + (\lambda + \lambda^2) z y = 0$$

or using the symbol I introduced in my treatise on the Gaussian series

$$P\begin{pmatrix} 0 & -\lambda & 0 & \\ 0 & 1+\lambda & 0 & z \end{pmatrix}$$

is that particular solution which for $z = 0$ is equal to 1.

According to the transformation principles developed in that treatise, y is not only expressed by the function $P(0, 2\lambda + 1, 0)$, but also by the functions $P(\frac{1}{2}, 0, \lambda + \frac{1}{2}$, $P(0, \lambda + \frac{1}{2}, \lambda + \frac{1}{2})$. One obtains from this a large number of representations of y using hypergeometric series and definite integrals, among which we note here only the following

$$y = F(1 + \lambda, -\lambda, 1, z) = (1 - z)^{\lambda} F\left(-\lambda, -\lambda, 1, \frac{z}{z - 1}\right)$$

$$= (1 - z)^{-1-\lambda} F\left(1 + \lambda, 1 + \lambda, 1, \frac{z}{z - 1}\right)$$

which suffices for all cases.

In order to derive results for Boyle's law, from those found for Poisson's law, one must reduce the values r, s, r', s' in Section 2 by $(a\sqrt{k})/(k - 1)$ and then let $k = 1$, by which one obtains $m = -1/2a$ and

$$v = e^{(1/2a)(r - r' + s - s')} \sum_{0}^{\infty} \frac{(r - r')^n (s - s')^n}{n!\,n!\,(2a)^{2n}}.$$

10

When one substitutes the expression for v found in the previous section into Eq. (4) of Section 8, one obtains the value of w for $r = r'$, $s = s'$ expressed in terms of the values of w, $\partial w/\partial r$, and $\partial w/\partial s$ on the curve s; but in our problem, only $\partial w/\partial r$ and $\partial w/\partial s$ are directly given along this curve and w must first be found from a quadrature of them. Thus, it is expedient to transform $w_{r',s'}$ in such a way that only the derivatives of w appear within the integral.

One designates by P and Σ the integrals of the expressions

$$-\mathrm{m}v\,ds + \left(\frac{\partial v}{\partial r} + mv\right) dr$$

and

$$\left(\frac{\partial v}{\partial s} + mv\right) ds - mv\,dr,$$

which due to the equation

$$\frac{\partial^2 v}{\partial r\,\partial s} + \frac{\partial mv}{\partial r} + \frac{\partial mv}{\partial s} = 0$$

are complete differentials, and designates by ω the integral of $Pdr + \Sigma\, ds$, which expression is also a complete differential due to the fact that

$$\frac{\partial P}{\partial s} = -mv = \frac{\partial \Sigma}{\partial r}\,.$$

One now chooses the constants of integration for these integrals so that ω, $\partial\omega/\partial r$ and $\partial\omega/\partial s$ vanish for $r = r'$, $s = s'$. Thus ω satisfies the equations $(\partial\omega/\partial r) + (\partial\omega/\partial s) + 1 = v$, $(\partial^2\omega/\partial r\,\partial s) = -mv$, and for both $r = r'$ and $s = s'$, $\omega = 0$; and is, incidentally, fully determined by the differential equation

$$\frac{\partial^2\omega}{\partial r\,\partial s} + m\left(\frac{\partial\omega}{\partial r} + \frac{\partial\omega}{\partial s} + 1\right) = 0.$$

Introducing $\omega_{r',s'}$ for v in the expression for ω, one can now transform it by partial integration into

$$w_{r',s'} = w_{c,r'} + \int_{c,r'}^{c,s'}\left(\left(\frac{\partial\omega}{\partial s} + 1\right)\frac{\partial w}{\partial s}\,ds - \frac{\partial\omega}{\partial r}\frac{\partial w}{\partial r}\,dr\right). \tag{1}$$

In order to determine the motion of the gas from the initial conditions, one must chose the curve c for which $t = 0$. On this curve $(\partial w/\partial r) = x$, $(\partial w/\partial s) = -x$, and one obtains by one more partial integration

$$w_{r',s'} = w_{c,r'} + \int_{c,r'}^{c,s'}(\omega\,dx - x\,ds),$$

consequently, according to Section 3, Eqs. (4) and (5)

$$(x - (\sqrt{\varphi'(\rho)} + u)t)_{r',s'} = x_{r'} + \int_{x_{r'}}^{x_{s'}}\frac{\partial\omega}{\partial r'}\,dx$$

$$(x - (\sqrt{\varphi'(\rho)} + u)t)_{r',s'} = x_{s'} - \int_{x_{r'}}^{x_{s'}}\frac{\partial\omega}{\partial s'}\,dx. \tag{2}$$

These Eqs. (2) only express the motion so long as

$$\frac{\partial^2 w}{\partial r^2} + \left(\frac{d \log \sqrt{\varphi'(\rho)}}{d \log \rho} + 1\right) t$$

and

$$\frac{\partial^2 w}{\partial s^2} + \left(\frac{d \log \sqrt{\varphi'(\rho)}}{d \log \rho} + 1\right) t$$

remain different from zero. As soon as one of these quantities vanishes, a condensation shock arises and then Eq. (1) is valid only within a range of values which lies entirely on one and the same side of the condensation shock. The principles developed here do not suffice, at least in general, to determine the motion from the initial conditions. However, one can determine the motion with the help of Eq. (1) and the equations which are, according to Section 5, valid for the condensation shock, as long as the position of the condensation shock at time t and thus ξ as a function of t is given. We do not wish to pursue this further, though and we also forego treating the case in which the air is bounded by a solid wall. There the calculation holds no difficulties and it is not at present possible to compare the results with experience.

*Theory of Multifoil Collision Supercompression**

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Translated from Russian by John Schoonover and Rachel Douglas

Investigations of compression flows, including relatively cold isentropic compression flowing like a Riemannian-centered wave, were begun quite some time ago[1,2,20] and in many respects were already completed with the work of Busemann's school.[3–5] High densities in the astrophysical range, however, were considered[6,10] separately from these studies. This was also the case for the possibility of performing experiments—until the discovery of methods for attaining unboundedly high densities.[17,19,21,24] It is true that earlier individual communications had appeared[7,8,12–16] similar to this new tendency; an idea of its status is given in Refs. 22–29, 31–38, 40, and 42–51. In particular, the potential "market" for research on superdense compression is discussed in Ref. 52. These investigations touch on such problems as attaining ultrahigh frequency matter oscillations in order to generate gravitational waves,[27] powerful pumping of hard coherent radiation,[33] laboratory simulation of stellar interiors,[21,47] as well as other extremal phenomena. The profusion and the promising nature of possible applications of ultrahigh compression justifies interest in studying this kind of compression itself—as a new, independent area of research.

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Already classical hydrodynamics[1-3,5] had deduced the law of infinite compression

$$\frac{1}{\langle \rho \rangle} \sim V \sim x(t) \sim \left(1 - \frac{t}{t_o}\right)^{2/(\gamma+1)} \rightarrow 0 \text{ as } t \rightarrow t_o \tag{1}$$

through the motion of a one-dimensional flat piston, which generates a centered, self-similar isentropic wave. A similar centered wave, composed of a fan of shock waves that converges to a point [see Fig. 2, (b) and (c), and Fig. 6] has been observed previously.[1,5,7,17,26,27] As long as the equation of state of an ideal gas is accepted

$$\frac{PV}{E} = \text{const.} = \gamma - 1 = \frac{2}{f} = \left\{\int_a^b \frac{T}{E}\, dS - \ln \frac{E_b}{E_a}\right\} \Big/ \ln \frac{V_b}{V_a}, \tag{2}$$

it follows that

$$P \sim \rho^\gamma \sim \left(1 - \frac{t}{t_o}\right)^{-2\gamma/(\gamma+1)} \rightarrow \infty \text{ as } t \rightarrow t_o, \tag{3}$$

which means that an unbounded mechanical external activation power[(1)] is needed.

$$\dot{E}_{pl} \sim \left(1 - \frac{t}{t_o}\right)^{-(3\gamma-1)/(\gamma+1)} \rightarrow \infty \text{ as } t \rightarrow t_o. \tag{4}$$

This characteristic "peaking" of the driver pulse[17-19,22-26,34-38,40,46-51] explains the characteristic profile of the activation power, but also serves as an obstacle to the attainment of high-strength compressions for rather varied reasons.[40,43,46] However, unbounded planar compression, Eqs. (1), (2), and (4), leaves constant the important parameter $\langle \rho R \rangle$, which increases under other compression schemes:

$$\langle \rho R \rangle_{pl}^{-} \equiv \rho L = \text{const.}$$

$$\langle \rho R \rangle_{\text{cyl}}^{\perp} \equiv \rho^{1/2} \left(\frac{M}{\pi L}\right)^{1/2}_{m/L-\text{const.}} \rho_{\text{s-const.}}^{1/2} \sim \left(\frac{E}{M}\right)^{3/4} \rightarrow \infty \text{ as } R_C \rightarrow 0$$

[(1)]The mechanical power of the external driver can differ substantially from the total power of the external driver,[19,49] and, moreover, from the power of secondary hydrodynamic processes. In the compression scheme we are considering here, the generation of power depends most importantly on the process of quasi-one-dimensional accumulation.

$\therefore \langle \rho R \rangle^{-}_{\text{cyl}} \equiv \rho L \to \infty$ with[(2)] $L = \text{const.}$

$$\langle \rho R \rangle_{\text{sph}} \equiv \left(\frac{3}{4\pi}\right)^{1/3} \cdot \rho^{2/3} M^{1/3}_{M-\text{const.}} \sim \rho^{2/3}_{s-\text{const.}} \sim \frac{E}{M} \to \infty \text{ as } R_s \to 0 \quad (5)$$

In view of the exceptionally stringent demands on the peaking regime for spherical—or in the majority of studies, quasi-spherical[7,44]—and cylindrical[11,13,17] compression, and on the homogeneity and symmetry of the driver,[28,36,40,46,49–51] compression schemes of the sort proposed in Refs. 27, 42, 47, and 49, with the elements of the compressing system initially dispersed, are of interest.

Later we will consider a compression scheme, an extremely simplified version of which reduces to the isentropic compression of a cylinder, when

$$R_{C^*} = \text{const.}, L \to 0 \therefore \frac{M}{L} \to \infty, \langle \rho R \rangle^{-}_{C^*} = \rho L = \text{const.}$$

$$\langle \rho R \rangle^{\perp}_{C^*} = \rho^{1/2}\left(\frac{M}{\pi L}\right)^{1/2} = \rho\left(\frac{M}{\pi \rho_{oo} L_{oo}}\right)_{M-\text{const.}}$$

$$\sim \rho_{s-\text{const.}} \sim \left(\frac{E}{M}\right)^{3/2}_{L \to 0} \to \infty \quad (6)$$

in contrast to Eq. (5) for the cylinder, and the growth of the transverse $\langle \rho R \rangle^{\perp}_{C^*}$ with the growth of ρ proceeds faster than in the spherical case. This means, in particular, that although the given value $\langle \rho R \rangle_{\text{sph}} \equiv \langle \rho_{oo} R_{oo} \rangle_{C^*} = \langle \rho R \rangle^{\perp}_{C^*}$ is reached as a result of the approximately identical mechanical (hydrodynamic) energy, that is $E_s \approx E_{C^*}$, since the thick disc differs insignificantly from a sphere, nevertheless, on account of relations $E_{C^*} \sim (\langle \rho R \rangle^{\perp}_{C^*})^{2/3}$ and $E_{\text{sph}} \sim \langle \rho R \rangle_{\text{sph}}$, compression of the cylinder along the axis spans a large range of values for $\langle \rho R \rangle$:

$$\langle \rho_{oo} R_{oo} \rangle^{\perp}_{C^*} < \langle \rho_{oo} R_{oo} \rangle_{\text{sph}} \xrightarrow{\leqslant} \langle \rho R \rangle_{\text{sph}} \equiv \langle \rho R \rangle^{\perp}_{c^*} \quad (7)$$

The question of the actual activation energy, as distinct from the "mechanical" energy that occurs in this treatment, is less trivial.

The scheme shown in Figs. 1, 3, and 4, proposed in Refs. 27 and 47, is quite similar to this type of compression: The elements of the target are initially dispersed and quite far from the point of maximum compression; they are accelerated "one-dimensionally" to a linear spatial

[(2)]Here M is mass, though later it will stand for Mach number; S is entropy.

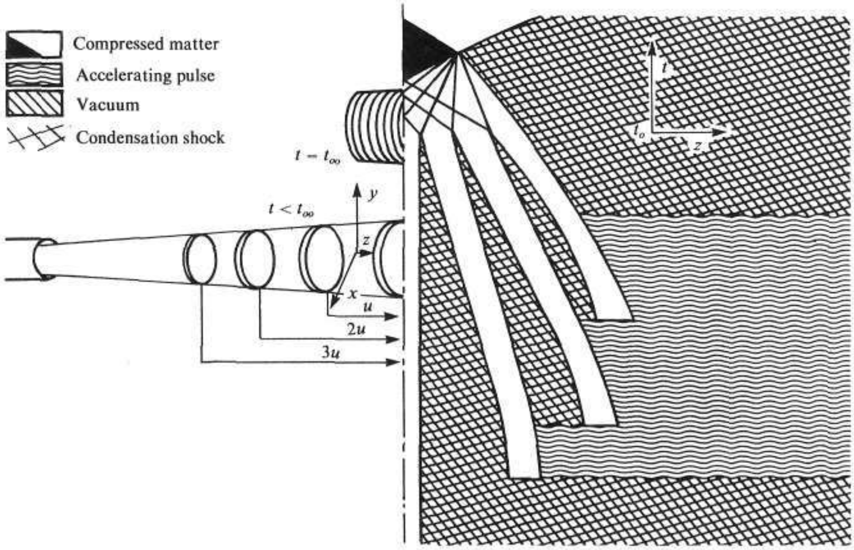

Figure 1. Schematic for a proposed thought experiment consisting of a packet of layers with neighboring layers approaching each other with the same velocity. The left half of the figure shows the layers moving toward each other. The right half shows several layers from the packet moving during a period of time. First, they are accelerated by an external driver pulse. They then reach a highly compressed state of matter, achieved by a series of condensation shocks. The whole process occurs in a vacuum.

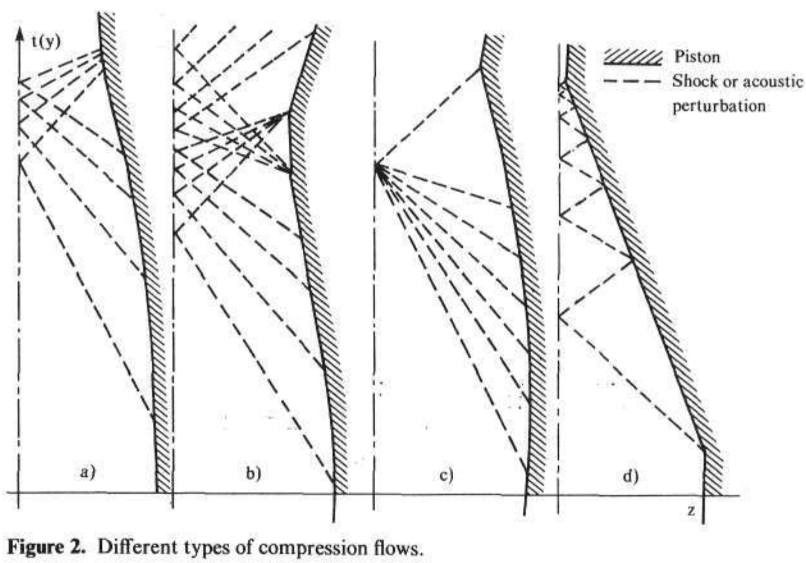

Figure 2. Different types of compression flows.

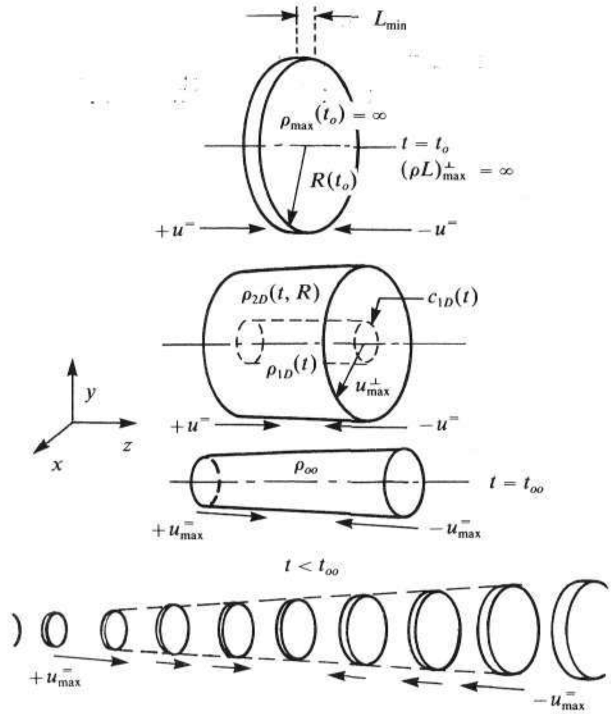

Figure 3. Ultrahigh compression scheme. Four stages in the compression are shown, starting with the earliest stage at the bottom of the figure and proceeding sequentially upward.

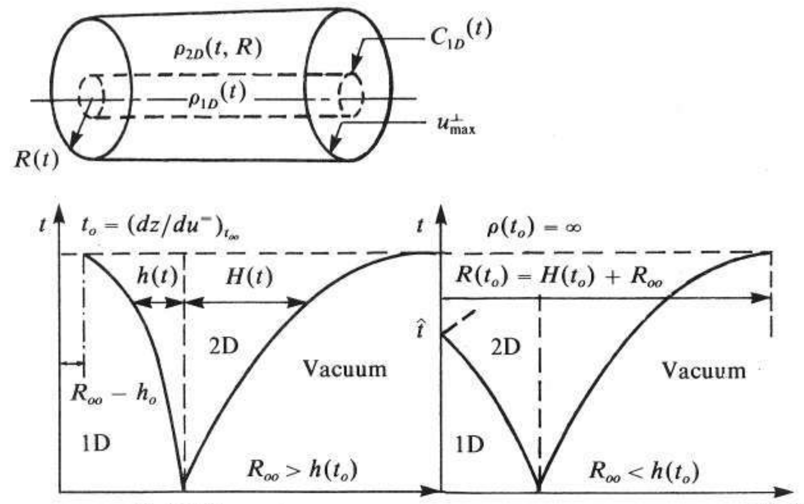

Figure 4. Graphic representation of the compression scheme shown in Fig. 3.

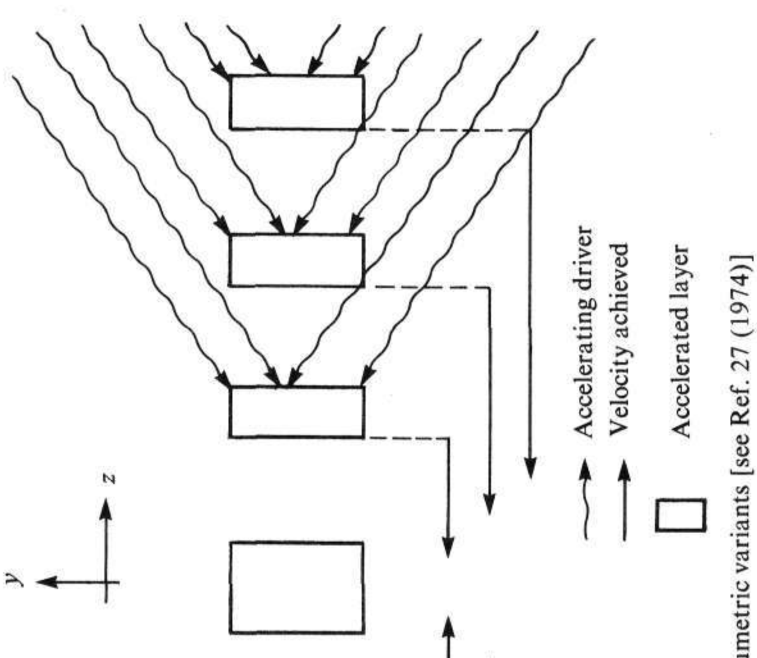

Figure 5. Alternate arrangement of layers for high compression. The middle layer is thicker than the others in this scheme.

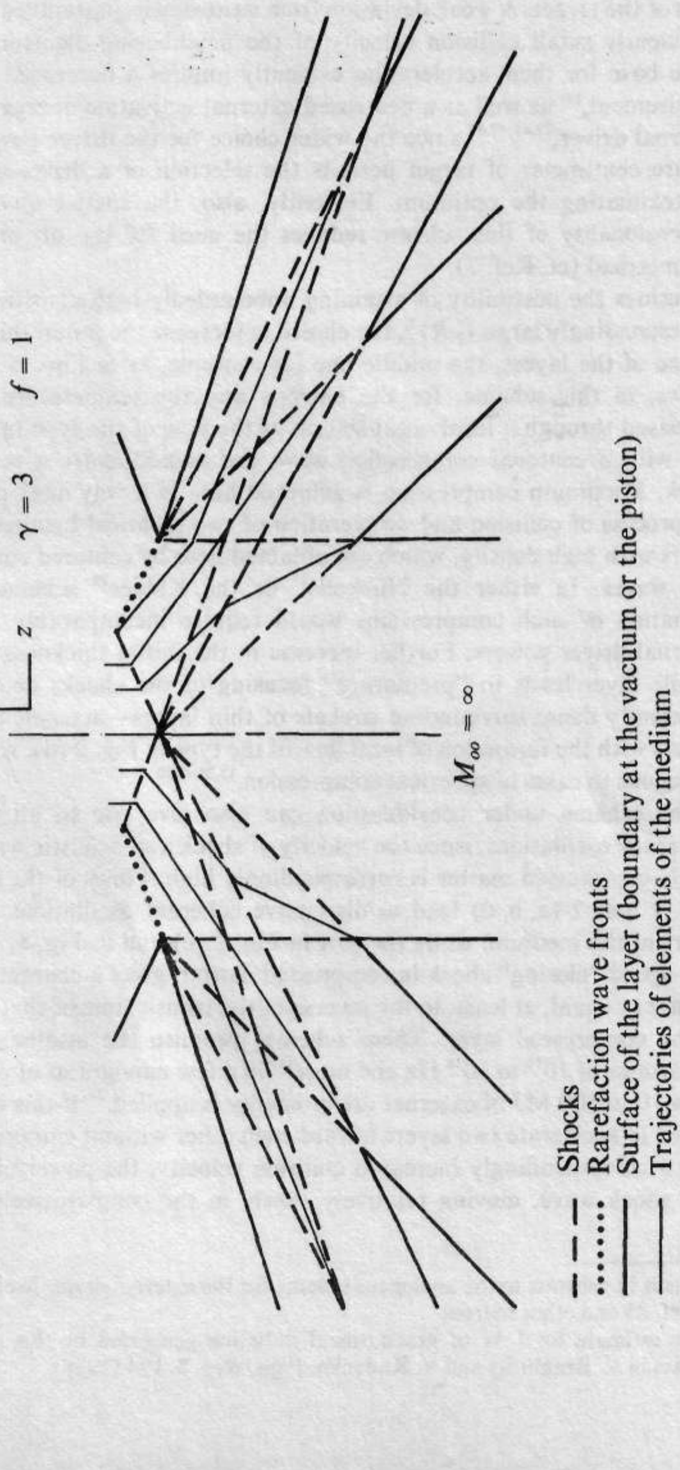

Figure 6. The effect of focused shocks in the scheme shown in Fig. 5.

distribution of velocities, up to the maximum values occurring at the ends of the target. A weak deviation from isentropy is guaranteed by the sufficiently small collision velocity of the neighboring elements. The large base for their acceleration evidently implies a decreased power requirement,(3) as well as a decreased external activation energy of the external driver,[27,45–47,49] since the wider choice for the driver power per square centimeter of target permits the selection of a driver scheme approximating the optimum. Evidently, also, the spatial quasi-one-dimensionality of this scheme reduces the need for the driver to be symmetrical (cf. Ref. 7).

Besides the possibility of attaining unboundedly high densities with correspondingly large $\langle \rho R \rangle^{\perp}$, the choice to increase the initial thickness of one of the layers, the middle one for example, as in Figs. 5 and 6, allows, in this scheme, for the entropy and the temperature to be increased through a local organization of the flow of the type in Fig. 2 (d), with a centered compression wave and consequently a reflected shock. Maximum compression is achieved here in a way analogous to the process of collision and deceleration of two identical homogeneous layers with high density, which are obtained here by centered compression waves. In either the Nuckolls[17] or the Kidder[18] schemes, the formation of such compressions would require incomparably higher external driver powers. Further increase in the initial thickness of the middle layer leads to "premature" focusing of the shocks or—given sufficiently dense surrounding packets of thin layers—acoustic disturbances with the formation of local flow of the type in Fig. 2 (d), which is analogous to cases of spherical compression.[17,29,49,50]

The schema under consideration can also give rise to ultra-high frequency oscillations, since the velocity of shock and acoustic waves in highly compressed matter is correspondingly high. Flows of the sort in Figs. 1 and 2 (a, b, d) lead to distinctive coherent oscillations of the layers of the medium, while the sort in Fig. 2 (c) and in Fig. 6, with a high-speed "closing" shock in compressed matter, gives a characteristic frequency equal, at least, to the inverse of the transit time of that shock in the compressed layer. These schemes promise the attainment of oscillations of 10^{16} to 10^{18} Hz and more(4) in a few nanograms of plasma when 10 to 100 MJ of external driver energy is applied.[27] If this energy is used to accelerate two layers toward each other without compression, with a correspondingly increased collision velocity, the powerful stopping shock wave, moving relatively slowly in the comparatively low-

(3)This is in contrast to the analogous scheme for the external driver itself given by Ref. 25 and other sources.

(4)Our estimate for 1 W of gravitational radiation generated by this process appears in V. Bzaginsky and V. Rudenko, *Phys. Rep.* **5**, 194 (1978).

density plasma, will lead to a significant increase in entropy; that is, to a frequency loss of many orders of magnitude.

The relativistic generalization of the results of the present discussion would be of interest—for example, the hypothetical case of simultaneous collisions of several nuclei, instead of the layers considered here, in the spirit of Ref. 30, with the attainment of ultranuclear densities.

Here we will confine ourselves to simple initial equations—nonrelativistic ones—without considering processes of continuous dissipation. Their applicability to two-dimensional processes of continuous dissipation. Their applicability to two-dimensional processes is *a priori* less obvious than to one-dimensional, as in Ref. 27, but they too give rise to far from trivial physical results—judging by the fact that the key concept of the new approach[17,47] is the use of an ideal hydrodynamic scheme for rapid, economical compression.

Turning to discussion of the concrete formulation of the problem and simple estimates, we will consider a packet of parallel layers—foils or some sort of "clots," equidistant, of identical thickness and composition, with identical speeds of approach to neighboring ones[27] Figs. 1, and 3. The collisions of layers, simultaneously throughout the whole packet, changes the speed of the medium only by Δu at the points of contact, without affecting the speed of the centers of mass of the layers until the arrival of the rarefaction wave from the edges. If the symmetry of the process is not disturbed, then, regardless of the equation of state, each interior layer is compressed, as with a two-way action by adiabatic pistons singly accelerated to Δu—with multiple reverberations of the shock waves and without the creation of rarefaction waves or of contact surfaces,[8,27] Fig. 2 (d). Only identical shocks interact, and the speed is changed only by these shocks there and only by the invariable constant amount Δu, equal to half the speed of approach of neighboring layers toward a collision.

Multiple reflections of the discontinuities generate nonlinear oscillations of the halflayers with an increasing characteristic frequency[(5)]

$$\nu \equiv \frac{\mathrm{D}}{L} = \nu_{oo}\left(\frac{D}{D_{oo}}\right)\left(\frac{\rho}{\rho_{oo}}\right) \equiv \nu_{oo}\left(\frac{D}{D_{oo}}\right)\Big/(1 - t/t_o), \tag{8}$$

where L is the half thickness of the layer, D is the shock velocity, which, after the k-th reflection is given by

$$D_k = \Delta u/(1 - \rho_k/\rho_{k+1}), \tag{9}$$

[(5)]This is a hydrodynamic approximation from below, since it ignores higher order harmonics, the existence of which is determined by the sharpness of the profile of the shock fronts, which depends on many factors.[2,7,9,14,26] with an evolution of the degree of compression

$$\rho_{1D}(0 \le t \le t_o)/\rho_{oo} = (1 - t/t_o)^{-1} \equiv (1 - t\Delta u/L_{oo})^{-1}. \tag{10}$$

The flow under consideration can be triggered not only by the collision of layers, but also by a piston, the speed of which increases, and subsequently can decrease by invariable, discrete amounts Δu. The corresponding two-dimensional stationary flows[27] are approximately analogous to this (cf. Refs. 3, 5, 7, and 18).

Being bound by the equation of state (2), we derive for any adjacent pair of discontinuities—incident and reflected—that distinguish three successive states "$k-1$," "k," "$k+1$" that are spatially homogeneous between the discontinuities, an invariable, discrete increase in the local velocity of the discontinuities:

$$D_k - D_{k-1} = \frac{\Delta u}{f} \equiv \frac{\Delta u(\gamma - 1)}{2} \therefore \frac{\Delta D}{\Delta u} = \pm \frac{(\gamma - 1)}{2} \equiv \pm \frac{1}{f} \tag{11}$$

Thus, since this flow can change velocity only by $\pm\Delta u$ with $D_k > D_{k-1}$ it is not difficult to see that along the discontinuities there exist nonisentropic invariants, analogous to Riemannian invariants[1–5,27]:

$$\frac{2D}{(\gamma - 1)} \pm u \equiv fD \pm u = B_{\pm} = \text{const.} \tag{12}$$

Given an arbitrary initial shock, the discontinuities weaken relatively monotonically:

$$\frac{D_k}{C_k} \equiv M_k = \{1 - 1/[1/(1 - 1/M_{oo}^2) + k/(f+1)]\}^{-1/2} \to 1 \text{ as } \frac{k}{f} \to \infty. \tag{13}$$

Thus, with the transition to the condition $k \gg f$, the solution approaches the isentropic case. One can now consider the leaps to the next equilibrium condition to be quasistatic, since they succeed in reestablishing the equilibrium, disrupted by the change of volume. On the other hand, the regime $k \lesssim f$, for which the velocity D to the establishment of the next equilibrium is almost equal to the speed Δu of its disruption, is clearly irreversible, because, for example, for $M_{oo} = \infty$, we have $M = 1 + (f+1)/k$.

We note that the complete "piston-wall" path of the shock is bounded from above for $f > 1$, that is for $\gamma < 3$, but is unbounded for $f = 1$, $\gamma = 3$.

This is analogous to the path of a molecule during the compression of a nonrelativistic gas composed of free molecules.

From Eq. (12) it immediately follows that for $\gamma = 3$ or $f = 1$, the laboratory velocities of the shocks are conserved:

$$\gamma = 3 \therefore D \pm u = \pm\left(\frac{dZ}{dt}\right)_{\text{discont.}} = B_{\pm}(f = 3) = \text{const.}, \tag{14}$$

from which the solution takes on an especially simple and apparent form, for example, as mentioned in the Doklady article cited in Ref. 27. In the process of "self-compression" of a packet of layers, focusing of the shocks appears toward its edges, due to the "stepwise-quasi-linear" distribution of velocities of the medium. The focusing of every k-th discontinuity to the point Z_φ at the instant t_φ corresponds to the relations

$$Z_\varphi = Z_k(t_{oo}) + (t_\varphi - t_{oo})\left(\frac{dZ}{dt}\right)_k = \pm D(t_\varphi - t_{oo}) \tag{15}$$

from which, taking into account Eq. (14), the equality

$$U(Z, t_{oo}) = \pm D_{oo} + (Z_\varphi - Z)/(t_\varphi - t_{oo}) \tag{16}$$

should be satisfied, which is seen to be due to the linear distribution of velocities of the medium

$$U(Z, t_{oo}) = -Z/(t_o - t_{oo});\, t_\varphi = t_o \tag{17}$$

at the points where the shocks originate. Upon the first contact of the layers, this distribution becomes continuous for the isentropic limit approaching when $D_{oo} \to C_{oo}$, $L_{oo} \to 0$.

Focusing at the edges is possible with $\gamma \neq 3$ too, and also at the center, by a correct choice of the thickness of the layers or of their composition. This gives rise to a "hot spot" (Figs. 2 and 6).

The nonisentropic analog of the Riemann invariant makes it possible to calculate, for instance, the maximum degree of adiabatic compression in the packet of layers, taking into account the total stopping of the flow at the center

$$\frac{\rho_{\max}}{\rho_{oo}} = \left(\frac{E_{\max}}{E_{oo}}\right)^{f/2} e^{-(f/2)\int_o^m (T/E)\, dS}$$

$$= \left[\left(1 + \frac{U_{\max}}{fD_{oo}}\right)\left(\frac{M_{oo}}{M_m}\right)\right]^f e^{-(f/2)\int_o^m (T/E)\, dS} \tag{18}$$

from which for $M_{oo} \gg \sqrt{f}$, if only the increase in entropy at the first strong discontinuity is taken into account, we have the maximum adiabatic compression

$$\frac{\rho_{\max}}{\rho_{oo}} \approx \left[\left(1 + \frac{U_{\max}}{fD_{oo}}\right)\Big/ M_{\min}\right]^{f}_{k \gg f} \approx (U_{\max}/fD_{oo})^{f} \equiv \left(\frac{\rho}{\rho_{oo}}\right)_{\Delta S - 0} M_{oo}^{-f} \quad (19)$$

Given an infinite speed at the edge of the packet, that is given an infinite "queue" of layers for a given initial gradient of their velocity in the z direction, the degree of compression can be infinite for the nonisentropic case as well. With a limited "queue" length equal to the thickness of the packet, the "cutoff" mechanism of the infinite compression in the one-dimensional flat approximation is a terminate rarefaction or splitting off wave, which transmits information to the center concerning the velocity at the edge. The "initial" speed of sound C_{oo} largely depends on the nature and intensity of the accelerating driver, which brings the layers to a spatially linear velocity distribution.

The most effective adiabatic compression, along the isentrope in local thermodynamic equilibrium, is achieved by weak shocks, the low amplitude of which can be ensured both by the choice of a sufficiently "dense" packet for a given initial gradient of the layers' velocity in the z direction, and, with a constant number of layers, by the lowering of $U_{\max}/C_{oo}$, that is of $\Delta U/C_{oo}$:

$$\left(\frac{\rho_{\max}}{\rho_{oo}}\right)_{\Delta S \to 0} \to [(\gamma - 1)U_{\max}/2C_{oo} + 1]^{2/(\gamma-1)} \equiv [U_{\max}/fC_{oo} + 1]^{f}. \quad (20)$$

To illustrate the influence of the composition of the layers on the compression process under consideration, we adduce two characteristic, although not too realistic, examples:

$$\begin{aligned} &\begin{matrix} f=\infty \\ \gamma=1 \end{matrix} : \left(\frac{\rho_{\max}}{\rho_{oo}}\right)_{\Delta s - 0} = \exp\{U_{\max}/C_{oo}\} \\ &\begin{matrix} f=1 \\ \gamma=3 \end{matrix} : \begin{matrix} \left(\frac{\rho_{\max}}{\rho_{oo}}\right)_{\Delta s - 0} = U_{\max}/C_{oo} + 1 \\ \left(\frac{\rho_{\max}}{\rho_{oo}}\right)_{\Delta s \gg Nk} \approx U_{\max}/D_{oo} \gg 1 \end{matrix} \end{aligned} \quad (21)$$

($\gamma = 1$ signifies isothermal compression with $\Delta s \ll 0$).

Furthermore, we estimate the minimum dimension occupied by the packet of layers at the stage of maximum compression:

$$\mathcal{L}^{\min}_{\Delta s=0} \cong \mathcal{L}_{oo} \Big/ \left(\frac{U_{\max}}{fC_{oo}}\right)^{f} \equiv fC_{oo} \Big/ \left(\frac{U_{\max}}{fC_{oo}}\right)^{f-1} \left(\frac{dU}{dZ}\right)_{oo} \overset{\left(\substack{f=1\\ \gamma=3}\right)}{}$$

$$= \frac{C_{oo}}{(dU/dZ)_{oo}} = \text{const}$$

$$\mathcal{L}^{\min}_{\Delta s \gg Nk} \cong \mathcal{L}_{oo} \Big/ \left(\frac{U_{\max}}{fD_{oo}}\right)^{f}_{(f=1)} = \mathcal{L}_{oo} \Big/ \left(\frac{U_{\max}}{D_{oo}}\right) \equiv \frac{D_{oo}}{(\Delta U/\Delta Z)_{oo}}$$

$$= \text{const.} \quad (22)$$

That is, the compression at the limit yields an infinitely thin layer for any specific heat except $f = 1$, $\gamma = 3$. In this case, building the packet up even to infinite velocity at the edge, for its given initial gradient along the packet thickness, does not change the minimum packet thickness.

Further, we will write down the maximum compression in a somewhat different form, which illustrates the weakening dependence of the maximum density on the initial conditions, as the density becomes higher:

$$\rho_{\Delta s=0} = (\rho^{\gamma}_{oo}/P_{oo})^{1/(\gamma-1)}(1/(f+2))^{f}(U_{\max} + fC_{oo})^{f}$$

$$\overset{(f=3)}{=} \left(\left[\frac{m_i}{m_e}\right]^{5/3} \frac{5m_e}{k^2}\left[\frac{8\pi}{3}\right]^{2/3}\right)^{5/2} \left(\frac{1}{5}\right)^{3} (U_{\max} + 3C_{oo})^{3}, \quad (23)$$

since in the given case, the thermodynamics, except for the density of the medium, is determined by the electronic properties of a completely degenerate Fermi-Dirac gas.

The one-dimensional results also allow us to estimate, with some precision, the two- and three-dimensional version of this compression process. Let us consider, as in section I of Ref. 53, the compression of a cylinder that simulates the results from the collision of a rather "thick" packet of foils, Figs. 3 and 4. The self-similar front of the rarefaction wave travels inward, and its path is $h(t)$. Depending on the heat capacity of the medium and the ratio $R_{oo}/h(t_o)$, two typical blow off-compression regimes can occur, Fig. 4. Thus, in the given isentropic case,

$$\rho_{1D}(0 \le t \le t_o)/\rho(0) = (1 - t/t_o)^{-1} \equiv \frac{1}{\tau} = \left(\frac{C(t)}{C_{oo}}\right)^{f}$$

$$h(t) \equiv \int_o^t c(t)\,dt = C_{oo}\int_o^{\tau} \frac{C(\tau)}{C_{oo}}\,d\tau \;\{(0 < f < 1; \gamma > 3) \rightarrow -\infty;$$

$$\frac{C_{oo}t_o}{1-1/f}(1-\tau^{1-1/f}) \rightarrow \begin{pmatrix} f>1 \\ \gamma<3 \end{pmatrix} \rightarrow \frac{C_{oo}t_o}{1-1/f} \equiv \frac{2C_{oo}t_o}{3-\gamma};$$

$$(f=1;\gamma=3) = C_{oo}t_o \ln\left(\frac{1}{\tau}\right) = C_{oo}t_o \ln\left(\frac{\rho_{\max}}{\rho_{oo}}\right) \rightarrow \infty \tag{24}$$

where the instant $\hat{t}$ of reflection of the self-similar wave front of rarefaction from the cylinder axis corresponds to:

$$t_o - \hat{t} = t_o\left\{1 - \left(\frac{3-\gamma}{2}\right)\left(\frac{R_{oo}}{C_{oo}t_o}\right)\right\}^{2/(3-\gamma)}_{(f-1,\gamma-3)}$$

$$= t_o \exp\left(-R_{oo}/C_{oo}t_o\right) \tag{25}$$

We estimate the outer edge of dispersion very roughly—using the Riemann invariant of the "instantaneously corresponding" state in the central one-dimensional region 1D in Fig. 4. Then, for $f>1$, $\gamma<3$ the blow off clearly lags behind the compression, that is, for $R_{oo} > h(t_o)$, $\langle\rho R\rangle^{\perp}_{t_o} = \infty$ is already achieved for region 1D, but for $R_{oo} < h(t_o)$ we obtain $\langle\rho R\rangle^{\perp}_{t_o} = \infty$ as a result of the retardation of the transverse expansion, since the isentropy guarantees the inequality $H < fh$, in accord with our use of the Riemannian invariant. In other words, even within the limits of such an exaggerated estimate of H, values for $\rho_{\max}$, $\langle\rho R\rangle^{\perp}_{\max}$ are obtained.

Numerical calculations confirm the qualitative result of this extremely crude estimate: The speed of dispersion grows monotonically in proportion to the transition to larger values of the parameter f, with corresponding behavior of $\langle\rho R\rangle^{\perp}$, the evolution of which reaches the one-dimensional asymptotic value, corresponding to the limit $R(t_o)$—as if it were the radius determined by inertia of an imaginary rigid pipe.

The unboundedness, in principle, of the values of $\langle\rho R\rangle$ achieved in the given scheme permit this scheme to be considered promising for the investigation of the possibilities of imitating stellar interior processes, in the spirit of Refs. 21 and 47, as well as other extremal phenomena.

We hope that the results derived here reflect new possibilities for the hydrodynamics of ideal liquids—"obviously" an extremely reduced and already exhausted descriptive method—to point out the regions of potentially interesting compression regimes, the further investigation of which demands different equations.[39,40]

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