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The Atomic Science Textbooks Don't Teach

The Significance of the 1845 Gauss-Weber Correspondence

by Laurence Hecht

The 1830's experiments of Carl Friedrich Gauss and Wilhelm Weber to test Ampère's electrodynamic theory, led to the conception of the electron and atomic nucleus, more than 50 years before their empirical confirmation.

A letter of 19 March 1845 from Carl Friedrich Gauss to his younger collaborator, Wilhelm Weber, ranks as one of the most singular interventions of all time by an individual in changing the course of history. Modern atomic science, physics, and chemistry, and everything in the modern world that depends upon them, would not have existed without it. It is, thus, one of the clearest proofs of the existence of the consumer fraud, which passes for university science education today, that the issues discussed in the letter are scarcely known to any but a few specialists today, and that not even one among these shows any adequate understanding of the fundamentals involved.

The point at issue in the cited correspondence, is the existence of a special form of scientific concept, known to Plato as the *Idea,* which had been introduced into electrodynamics by André-Marie Ampère some 20 years earlier. No other scientist in the world at the time recognized the significance of this aspect of Ampère's work. Gauss, in the 1845 letter, points to precisely this, and successfully provokes a reorientation of Weber's thinking. As a result, Weber develops a generalization of Ampère's law that leads, by no later than 1870, to the theoretical recognition of the existence of the charged atomic nucleus and oppositely charged orbiting electrons, decades before any empirical identification of the phenomena could be made. By that year, Weber had also derived the precise formula (e^2/mc^2) for the atomic measurement later known as the *classical electron radius,* and identified the nuclear binding force, a phenomenon for which there was no empirical evidence until the 20th century.

The fact that these discoveries of Weber are virtually unknown today, is itself a scandal, although not the main point of our treatment here. We focus rather on the more crucial underlying point: the method of Ampère, Gauss, and Weber; that is, the actual scientific method, which alone leads to fundamental discovery. The 1845 correspondence offers a precious inside view into the process.

First, the essential background:

In 1820, Hans Christian Oersted first demonstrated the effect of an electrical current on a magnet (Figure 1). Biot, Savart,

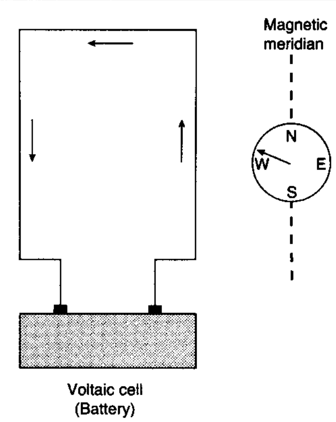
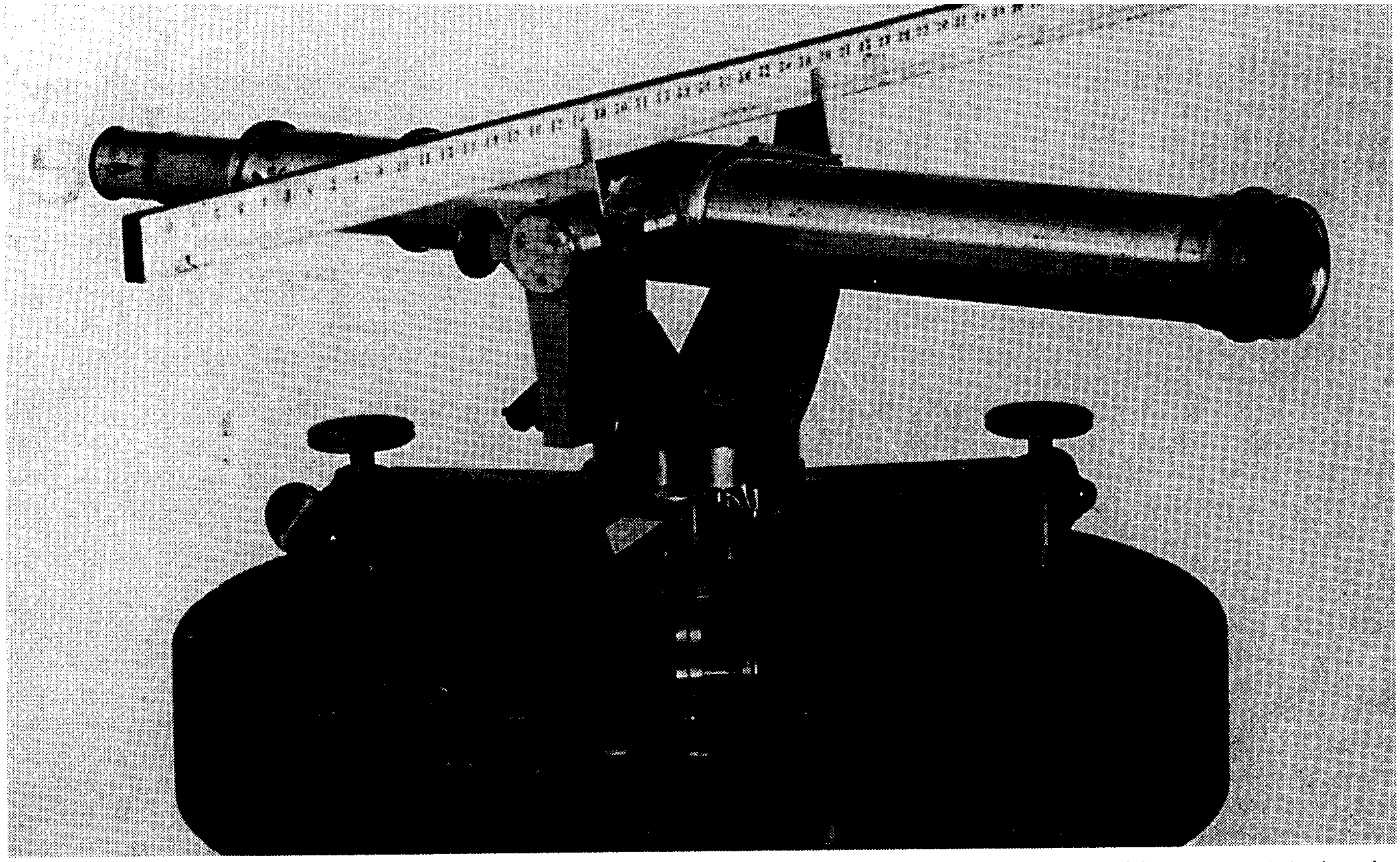


Figure 1
THE OERSTED EXPERIMENT
Hans Christian Oersted first definitively demonstrated a relationship between electricity and magnetism. He showed that a magnetic compass needle, placed near a current-carrying wire, will turn in the direction of the wire.



This telescope and meter stick were built by the firm Utzschneider und Fraunhofer of Munich, probably in 1880. It is, in principle, the same as that devised by Gauss in 1832 for the precise observation of angular deflection in connection with his determination of the absolute intensity of the Earth's magnetic force. A plane mirror, attached to a rotatable magnet, projects the image of whatever part of the meter stick it faces, into the telescope tube. The scale numbers are thus mirror-reversed and inverted for reading through the telescope sight. Weber's 1841 version of the apparatus could produce an angular precision of about 18 seconds of arc. (See Figure 1.3, page 37 for details of the Spiegel und Fernrohr *apparatus.)*

and others among the leading establishment physicists in France, encouraged by Laplace, undertook empirical investigations to determine the measurable effect of the conducting wire on a magnet. Ampère, recognizing that current (Galvanic) electricity represented a completely new phenomenon, saw in Oersted's demonstration the possibility of gaining fundamental new knowledge of magnetism and the atomic constituency of matter. Hypothesizing that magnetism itself may be the result of electrical currents surrounding the molecules of matter, he set out first to determine if two electrical conductors affected each other in the same way that a single electric wire affects a magnet. His first experiments established that two parallel conducting wires attract or repel, depending on whether their currents flow in similar or opposing directions. He next demonstrated that, by passing a current through a helically coiled wire, the configuration, which Ampère first named a solenoid, developed north and south magnetic poles, just like a bar magnet (Figure 2).

Having thus discovered, within a few weeks' time, the first empirical laws of a new science—he named it *electrodynamics*—Ampère next set himself the task of determining its fundamental laws. He thus proposed to find a formula expressing the interaction between two hypothetical, very small portions of electrical current, which he called *current elements,* in adjacent conducting wires. His results were sensational. At the time, the laws of gravitation, static electricity, and magnetism had all been found to be dependent on the inverse square of

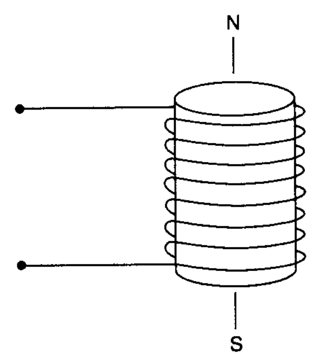


Figure 2
THE AMPERE SOLENOID
Ampère hypothesized that the true cause of magnetism is the motion of resistance-less electrical currents in tiny orbits around the molecules of matter. To prove it, he constructed the world's first electromagnet, a conducting wire coiled around a cylinder, which he named a solenoid. When the solenoid is attached to a battery, the ends of the cylinder become like the north and south poles of a bar magnet. Ampère believed that the large-scale circular motion of the electricity in the solenoid coil mimicked the tiny circular orbits which he conceived to be present in a magnet.

the distance of separation of their elements (mass, charge, and magnetic *molecules*). But Ampère's law of force for current elements showed a dependence not only on the distance, but on the directions of the current elements.

The Method of Hypothesis

The existence of such an anomaly, defying the neat unification of forces only recently established, was disturbing to many. For more than 20 years, Ampère's work, although well known to scientists, was never treated seriously. Although many criticized it, no one before Weber ever troubled to test it. The essential problem militating against its acceptance, was the philosophic outlook known as *empiricism*. A prevailing view in science then, as now, empiricism demanded that no physical phenomena could be measured, and thus subjected to the rigorous mathematical analysis expected of the pure sciences, unless one could see, hear, feel, smell, or taste it.

The method of hypothesis employed by Ampère, assumes, rather, that the so-called *data* of the senses are completely delusional. Nothing that one can see, hear, feel, smell, or taste is what it appears to be. Take a simple object like a magnetized steel bar, for example. An empiricist might measure and analyze the effect on it of an electrical wire all day long; he might cut it in half, grind it up into a powder, dissolve it in acid, or melt it in an oven, and never yet arrive at the simple *hypothesis* that its magnetic property derives from the existence of very small, electrical currents circling the invisible particles which constitute it.

But to merely formulate such an idea, is only the first small step in the pursuit of the method of hypothesis. It is necessary, above all, to seriously *believe* in the existence of such non-observable things. One must have a truly passionate belief, not unlike the proper meaning of the word *faith,* in the *reality* of such a mere idea. Only by such a driving passion, a *love of the idea,* can a person be motivated to pursue it, as Ampère did, through five years of experimental design and mathematical analysis, before he felt sure of its truth. And if the *idea* is of a fundamental sort, as was Ampère's concept of electrical action, it will tend to overthrow previously existing conceptions. In this case, the existence of a force dependent on angular relationship, clearly challenged the Newtonian conception. The laws of physics would not allow it—and yet it existed.[1]

Gauss appreciated Ampère's accomplishment as few, if any, others did. His letter of 19 March 1845 focusses on an aspect of Ampère's hypothesis, that is an Idea, known as the *longitudinal force.* This is a simple construct, relying only on elementary relationships of geometry, but so controversial that many have denied its existence for almost two centuries. As an understanding of it is crucial for the rest of the story, let us summarize how Ampère develops it.

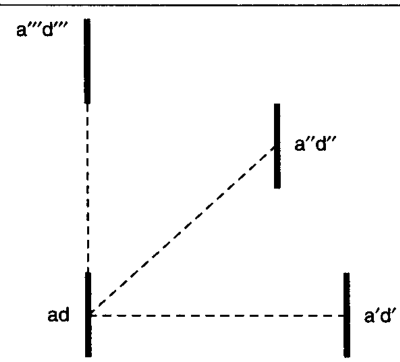


Figure 3
THE FORCE BETWEEN CURRENT ELEMENTS
Early experiments with two parallel wires showed that the wires attracted each other when the current flowed through them in the same direction, and repelled in the opposite case. From this Ampère could conclude that any two parallel, small sections of the wire (current elements) would behave accordingly. This is the relationship of element ad to a'd' in the diagram. But what if the second element is in another position, such as that of a''d'' or a'''d'''? Direct observation could not decide these more general cases.

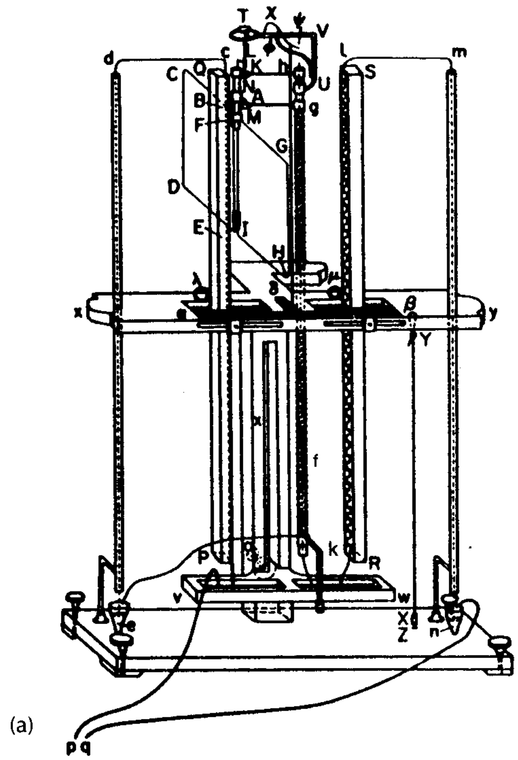
1. The Essentials of Ampère's Law

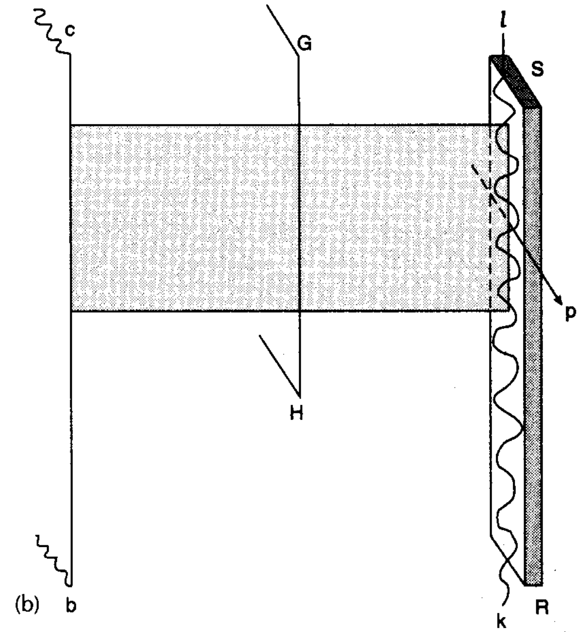
Consider first, two current elements, *ad* and *a' d'*, parallel to each other and perpendicular to the line connecting their midpoints (Figure 3). Ampère knew from his first experiment with parallel wires, that the current elements will attract or repel depending on whether the current in the wires of which they are a part flows in the same or opposite directions. But what about the current elements in other positions, such as *a''d''* or *a'''d'''*? How does the force between them differ when the second current element is positioned *longitudinally,* that is, on a straight line with the first, as at *a'''d'''*? Let the ratio of the force between the current elements in the longitudinal position to those which are parallel be designated by the constant *k*. What is its value? Two current elements cannot be isolated from the circuits of which they are a part, to be placed in these positions. Thus it is not possible to carry out a direct empirical measurement of the force between them. The method of hypothesis is the only one available to answer the question. Here is how Ampère proceeds in what is known as his Second Equilibrium Experiment:

Two parallel, vertical columns are placed a small distance apart on a laboratory table. Between them, a rectangular wire circuit is placed so that one side of the rectangle is parallel to the two columns and forms a common plane with them. The rectangle is free to swing on a vertical axis (Figure 4). In the first case, straight conducting wires are led up each of the vertical columns, and a current is made to flow through both of them in the same direction, either up or down. A current of the opposite direction is caused to flow through the parallel side of the rectangle *GH*. When the current is turned on, he finds that the rectangle remains positioned in the center between the two columns, being equally attracted or repelled by the two parallel wires in the vertical columns.

In the second case, the wire, *kl,* running up one of the columns, *RS,* is made to snake arbitrarily back and forth in the plane perpendicular to the paper—Figure 4 (b). But when the current is turned on, it makes no difference. The side, *GH,* of the movable rectangle still positions itself in the center between the two columns, and does so even when the pattern of bends in the wire *kl* is changed.

To explain this paradox, Ampère analyzes the current flow in the bent wire into its components in the vertical and horizontal direction. Both the bent and straight wires stretch along the same vertical length, hence the sum of the vertical components of *kl* is the same as *bc*. As the first case shows that no





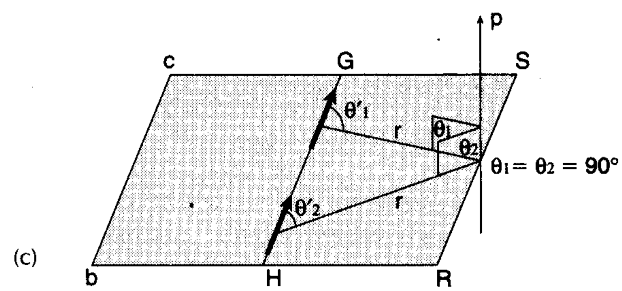


Figure 4
AMPERE'S SECOND EQUILIBRIUM EXPERIMENT

Ampère's most important law of interaction of current elements can be deduced from his Second Equilibrium Experiment. The copperplate (a) of the apparatus is from Ampère's 1826 Memoir. The two wooden posts PQ, RS are slotted on the sides which mutually face each other. A straight wire runs up PQ, while the wire in the slot of RS snakes back and forth in the plane perpendicular to PQRS. The wire rectangle CDGH also conducts a current and is free to rotate about the axis MI.

The purpose of the experiment is to determine whether the snaking of the wire in RS causes a rotation of the wire rectangle CDGH. The circuit is arranged so that current flows up the two fixed conductors in PQ and RS and down the side of the movable rectangle denoted GH. The entire apparatus is a single circuit. Current enters at the mercury-filled trough v, and leaves through the mercury cup at n. The wire passing up the vertical glass tube fgh is wound helically to negate its magnetic effect in a lateral direction. The vertical columns de and mn are glass tubes for the return circuit.

In (b) we see a schematic detail of the relationship of the two fixed conductors and the side GH of the movable rectangle: p is the horizontal component of current flow at an arbitrary point of the snaky wire kl. The shaded rectangle depicts the plane RScb to which p is perpendicular.

In (c), the rectangle RScb has been rotated 90 degrees so that the current element p now appears vertical. The midpoints of the two arbitrary current elements depicted in the plane RScb are connected by the lines r to the midpoint of p. The angle formed by r at p (θ_1, θ_2) is always right. The experiment shows that regardless of the other angle (θ'_1, θ'_2), the current element p exerts no force on any current element in the plane RScb.

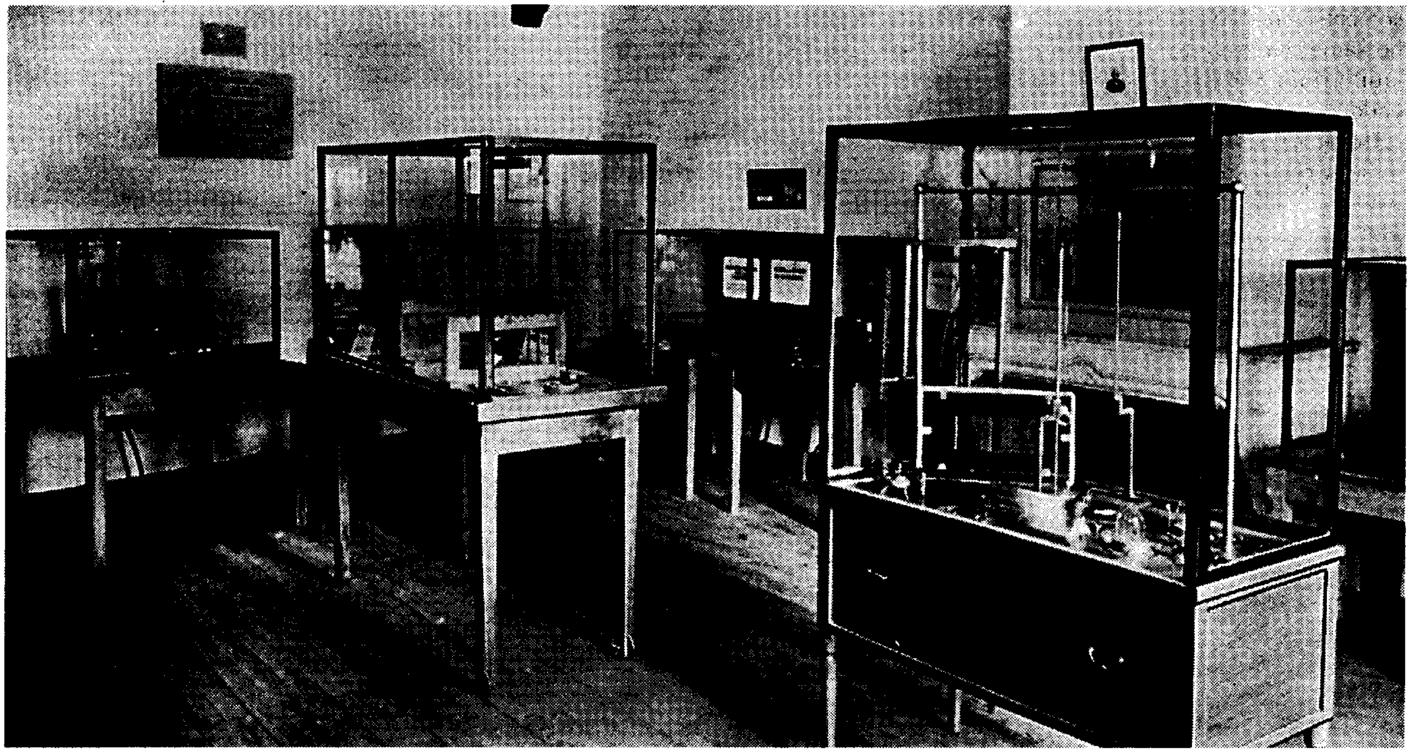
motion arises from two parallel vertical wires, the vertical components of *kl* may thus be ignored. Thus the problem reduces to an examination of its horizontal components.

Take any horizontal component, for example the one at the arbitrary position *p* in Figure 4(b), which is, by definition, perpendicular to the shaded plane *RScb*. Consider its relationship to any current element in the movable wire *GH*. By virtue of the fact that *GH* does not move, we can conclude that the arbitrarily chosen horizontal component must have no interaction with any current element along the length of *GH*. If it had any interaction, a disequilibrium of forces would necessarily arise from the arbitrary bends in the wire, causing *GH* to move.

From this experimentally deduced fact, Ampère is able to adduce the following theorem for the interaction of current elements:

> . . . that an infinitely small portion of electrical current exerts no action on another infinitely small portion of current situated in a plane which passes through its midpoint, and which is perpendicular to its direction. [Ampère 1826, p. 202]

Figure 4(c) helps us to see how the generalization is made. The plane *RScb* has been rotated 90 degrees. The horizontal component is here pictured as the arrow, *p*, passing vertically through the plane. The other two arrows represent arbitrary



Ampère's electrical apparatus is on display at a small museum in his childhood home at Poleymieux, France.

current elements in *GH*. These may make any angles whatsoever with the lines *r* connecting them to the midpoint of *p*. Since the experiment shows that the current element *p* exerts no action on any of them, the generalization is made that *p* exerts no action on any current element anywhere in the plane.

One might first imagine that we could establish the same result by simply placing two wires perpendicular to each other, a certain distance apart, and directly measuring the effect. The problem is that it is only the infinitesimal elements of the two wires, precisely at the point of perpendicularity which concern us, while in the mooted simplified configuration, all the other elements of the two wires will also contribute to the measured effect. One sees then, the genius of the Second Equilibrium Experiment, that it allows us to isolate—although only by abstraction—precisely the effect we wish to measure. This is true, indeed, of all four equilibrium experiments; the reader should know that Ampère carried out many dozens, perhaps hundreds, of other experiments before being able to reduce his presentation to deduction from the four equilibrium experiments presented in his final 1826 *Memoire* on electrodynamics.

The theorem deduced from the second experiment is the key that allows Ampère to solve the problem posed in connection with Figure 3. Return to Figure 3, where *ad* and *a'd'* are two parallel current elements. The question he poses is how the force between them changes as the current element is repositioned from *a'd'* to *a'''d'''*. Ampère determines that he will define the force as a function of the current element lengths, the intensities of the current of which they are part, and their relative position; the force is to be represented as acting along the line *r*.

Since the current elements under consideration lie in a plane, their relative position will be completely described by the length of the line, *r*, connecting their midpoints, and the angles θ, θ' which they form with it—Figure 5(a). "Consideration of the diverse attractions and repulsions observed in nature," writes Ampère,

> led me to believe that the force which I was seeking to represent, acted in some inverse ratio to distance; for greater generality, I assumed that it was in inverse ratio to the *n*th power of this distance, *n* being a constant to be determined. [Ampère 1826, p. 200]

If the very small lengths of the current elements are represented as *ds, ds'*, their intensities as *i, i'*, and θ, θ' designate the angles they form with the line connecting them, then the force between them, based on the assumptions so far, will be

$$\frac{ii' \; ds \cdot ds'}{r^n} \; \phi(\theta, \theta') ,$$

where φ represents the unknown function of the angles between the two current elements.

This leaves two unknowns to be determined: the value of the exponent, *n,* and the angle function, φ. The results of the second equilibrium experiment make it an easy matter to find the angle function φ. Take two arbitrary current elements in a plane, *ds* and *ds'*, and resolve their directions into two perpendicular components, as pictured in Figure 5(b). The parallel components will be represented by *ds* sinθ and *ds'* sinθ'. The longitudinal components will be represented by *ds* cosθ and *ds'* cosθ'. By the theorem derived from the second equilibrium experiment, we see that the force between *ds* sinθ and *ds'* cosθ', and also that between *ds'* sinθ' and *ds* cosθ is zero. This may appear confusing at first, because the two perpendicular elements under consideration in Figure 4 are not, in general, in the same plane. But the theorem deduced from the Second Equilibrium Experiment subsumes the planar case, as the reader can see from Figure 5(c).

The action of the two elements *ds* and *ds'* therefore reduces to the two joint remaining actions, namely the interaction between *ds* sinθ and *ds'* sinθ', and between *ds* cosθ and *ds'* cosθ'. It is easy to see that these two pairs of actions are between components which are either *parallel* or *longitudinal.* The first can be represented as

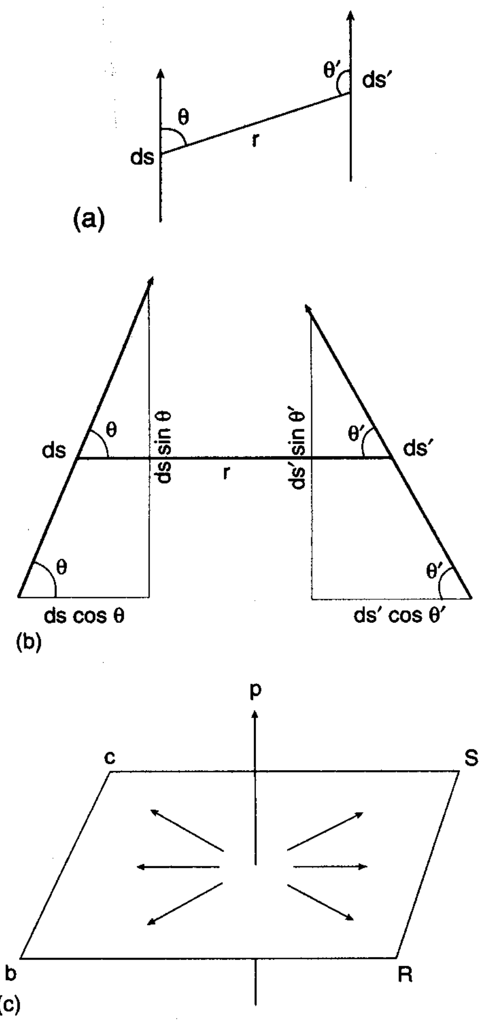


Figure 5
DEDUCTION OF THE AMPERE FORCE LAW
In (a), two parallel current elements ds, ds' form the angles θ, θ' with the line r joining their midpoints.

The horizontal and vertical components of two arbitrary current elements ds and ds' are pictured in (b). These are ds sinθ, ds' sinθ', dscosθ, and ds' cosθ'. The theorem deduced from the Second Equilibrium Experiment allows the elimination of two of the interactions (ds cosθ · ds' sinθ' and ds' cosθ' · ds sinθ).

In (c) are depicted current elements in the plane RScb, which also form a common plane with the perpendicular p.

$$\frac{ii' \; ds \cdot ds' \; \sin\theta \sin\theta'}{r^n},$$

and the second as

$$\frac{ii' \; ds \cdot ds' \; k \cos\theta \cos\theta'}{r^n},$$

remembering that *k* represents the ratio of the longitudinal to the parallel force, taking the parallel force as unity. It is only necessary to add these to obtain the total force between the two current elements, which produces:

$$\frac{ii' \; ds \cdot ds'}{r^n} (\sin\theta \sin\theta' + k \cos\theta \cos\theta') . \qquad \textbf{Eq. 1}$$

With only one simplification, introduced to ease the reader's burden, this is the general expression for the Ampère force under discussion in the 1845 correspondence between Gauss and Weber. For simplicity's sake, we derived the formula for the plane only. If the two current elements are not restricted to a plane, but may lie in planes whose angle with each other is represented by ω, then the full expression for the Ampère force becomes:

$$\frac{ii' \; ds \cdot ds'}{r^n} (\sin\theta \sin\theta' \cos\omega + k \cos\theta \cos\theta') .$$

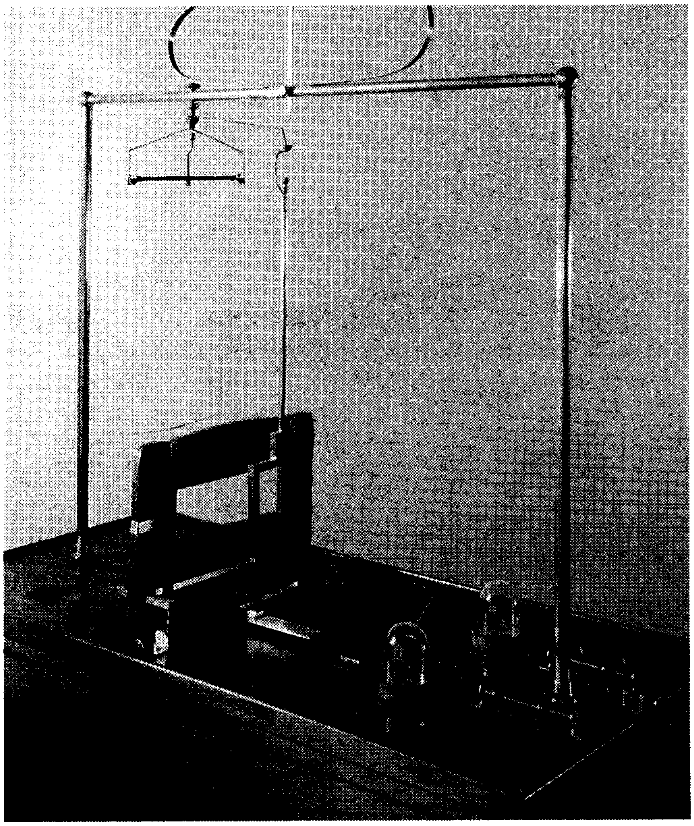
The determination of the values of the constants *n* and *k*, required two additional equilibrium experiments, which allowed Ampère to derive the values $n = 2$ and $k = -1/2$.

2. The Ampère Formula and the Correspondence

In 1828, Wilhelm Weber, a young physics graduate who had distinguished himself through original research into acoustics and water waves, met Carl Friedrich Gauss, then the leading astronomer and mathematician of Europe, at a scientific conference in Berlin. Gauss needed help to carry out the researches he planned in magnetism and electricity, and Alexander von Humboldt encouraged their cooperation. Weber was awarded a professorship at Göttingen University and began work there in 1831. Their joint researches on magnetism led to the first determination of an absolute measurement of the Earth's magnetic force and a seminal paper by Gauss on the subject in 1832. In 1833, the two constructed the world's first electromagnetic telegraph, running from the university observatory to the physics laboratory. Gauss had identified the confirmation of Ampère's law as one of the leading tasks facing science, and Weber began a long series of experiments to that end. The Gauss magnetometer was adapted into an instrument, the electrodynamometer, for bringing to electrical measurements the precision which Gauss had achieved for magnetism. (See accompanying article, p. 35.)

In 1845, Weber, now at Leipzig, was preparing a treatise on his results, which he wished to present to the Royal Society in Göttingen. Uncertain of his conclusions, he sent a copy to Gauss on 18 January 1845, asking for his evaluation. On 1 February 1845, Weber sent a second letter explaining a change he had made in the Ampère formula, Deutsches Museum

This experimental apparatus for replicating the electrodynamic experiments of Ampère was designed by H. Pixii in 1824, and is on display at the Deutsches Museum in Munich.



which I seek to justify, by means of the consideration that the empirically derived definition of the coefficient of the second term, which I have discarded, seems completely untrustworthy, because of the unreliability of the method, and hence that coefficient, so long as it lacks a more precise quantitative determination, by the same reasoning would have to be set = 0.

The coefficient Weber refers to is that identified just above as k. The second term, $k \cos\theta \cos\theta'$, is the longitudinal force, which Weber proposes to drop. It is perhaps not irrelevant that in the year 1845, an article by Hermann Grassmann in the physics journal, Poggendorf's *Annalen,* challenged the angle dependency of the Ampère force, describing the existence of such an effect as "improbable." Weber was a friend of the journal's editor, Poggendorf, and had recently worked with him in Berlin. Weber would thus have likely known in advance of Grassmann's contribution on the topic that had occupied him for more than a decade. Perhaps Grassmann's effort, combined with his separation from Gauss, propelled Weber into self-doubt about the reality of the Ampère hypothesis?

Gauss's rejoinder of 19 March is the singular intervention referred to in opening this article. In his 70th year, Gauss begins with regrets over the loss of time caused by his poor health, and his decade of removal from work on the topic. But, of the proposed modification of the law, Gauss writes, with no loss of acuity:

> . . .I would think, to begin with, that, were Ampère still living, he would decidedly protest . . . [I]n the present case, the difference is a vital question, for Ampère's entire theory of the interchangeability of magnetism with galvanic currents depends absolutely on the correctness of [his formula] and is wholly lost, if another is chosen in its place.
>
> . . . I do not believe that Ampère, even if he himself were to admit the incompleteness of his experiments, would authorize the adoption of an entirely different formula, whereby his entire theory would fall to pieces, so long as this other formula were not reinforced by completely decisive experiments. You must have misunderstood the reservations which, according to your second letter, I myself have expressed. . . .

To see clearly what Gauss is saying, the reader must know that prior to Ampère, magnetism had been explained as a separation (polarization) of two *magnetic fluids,* boreal and austral, within the particles (*magnetic molecules*) of a magnetizable substance. Magnetizing an iron bar was seen to consist of polarizing and aligning the magnetic molecules along a given axis of the bar. Ampère suggested rather that the magnetic molecule is an electrical current loop, a *"galvano-electric orbit,"* as Gauss was to characterize the Ampère magnetic hypothesis in his 1832 study of magnetism. Magnetization, for Ampère, consisted of aligning these microscopic current loops along the magnetic axis. In his 1826 treatise, Ampère had elaborately developed the interdependence of his new magnetic hypothesis with his formula for the force between two current elements.

Weber completely accepted Gauss's correction and wrote back on 31 March:

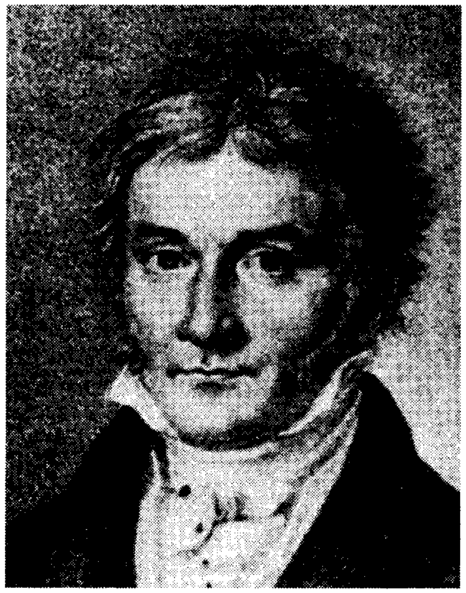
> It has been of great interest to me to learn from what you were kind enough to write, that Ampère, in the definition of the coefficient he calls k in his fundamental law, was guided by other reasons than the ones from immediate empirical experience which he cites at the beginning of his treatise, and that hence the derivation, which I first gave, because it seemed somewhat simpler, is inadmissible, because it does not reproduce Ampère's law with exactness; yet, by means of what seems to me to be a slight modification in my premise, I have easily obtained the exact expression of Ampère's law.

3. The Development of Weber's Law

Over the previous 10 years, Weber had been engaged in the experimental confirmation of Ampère's law. The measuring instrument he had developed, the *electrodynamometer,* consisted of a fixed and a rotatable helically wound electromagnet. (See accompanying article, p. 35.) The rotatable one, suspended by two wires whose torsion could be accurately measured, came to be known as the *bifilar coil.* A precisely measured current was passed through the two coils, and the angle of rotation observed by means of a precision system developed by Gauss for his magnetometer, consisting of a mirror and telescope. The effect of the Earth's magnetic force could be precisely determined, and thus eliminated, using the system already developed by Gauss. Hence the experimental

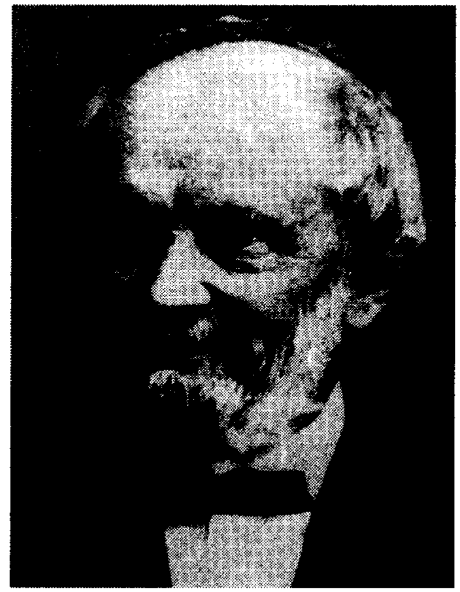
Courtesy of the Museum of Electricity at Poleymieux

André-Marie Ampère (1775-1836)



Lithograph by Siegfried Bendixen, courtesy of Historical Collection of the Göttingen University I. Physical Institute

Carl Friedrich Gauss (1777-1855)



E. Scott Barr Collection, American Institute of Physics Emilio Segrè Visual Archives

Wilhelm Eduard Weber (1804-1891)

data could be reduced to yield the exact rotational moment exerted between the two Ampère solenoids.

Ampère had already shown in his famous treatise (Ampère 1826), how to calculate the rotational moment exerted by a single circular current loop on a current element in any position in space. The force was dependent on the current strength, the distance of separation, and the area enclosed by the loop, all values which were determined in the Weber apparatus. By treating the helically wound coils as a compound of such current loops and integrating their effect, Weber was able to calculate with precision the angular rotation that should be imparted to the bifilar coil. His measurements, achieved under a variety of experimental conditions, conformed to the calculated values within one-third of a scale unit, or less than 6 seconds of arc.

Despite this complete agreement between theory and measurement, which Weber had already determined before 1845, there had remained the possibility that the Ampère law was not correct in all its specificity, and that a simpler generalization, discarding the longitudinal force, might suffice. After receipt of Gauss's 19 March letter, with full confidence in the master's judgment, Weber forged ahead into new territory.

The task he set himself was to find a generalization of Ampère's law that would encompass the phenomena of voltaic induction, discovered by the American, Joseph Henry, five years after Ampère had completed his work in electrodynamics. This included the following effects:

• the appearance of an electrical current in a closed circuit when there is relative motion between it and a current-carrying wire in its vicinity.

• the appearance of an electrical current in a closed circuit when there is a change in the intensity of current in a neighboring conductor.

Ampère's electrodynamic law applied only to moving currents in fixed conductors. Beside it, stood the separate law of electrostatic force. The existence of the phenomenon of induction suggested to Weber that a true, fundamental law of electricity would have to subsume the electrostatic and electrodynamic laws under a new, more general form. A conception developed by his colleague, Gustav Fechner, proved to be of crucial value.

Fechner had extended the Ampère conception of the current element by considering the flow of electricity as consisting of oppositely charged electrical particles moving through the conductor with equal velocity in opposite directions. (Today, we assume that the positive electrical particle is virtually stationary and that the negative particle moves, a modification first suggested by Wilhelm Weber.) In any small segment of the wire, a positive and a negative particle would be found speeding past one another. Thus, the interaction between two current elements involved four interactions among electrical particles. If the current elements are labeled e and e', there are the following four relationships:

(1) between $+e$ and $+e'$
(2) between $+e$ and $-e'$
(3) between $-e$ and $-e'$
(4) between $-e$ and $+e'$.

Since the particles, in these cases, are confined to their conductors, the forces between them are assumed to be transferred to the motion of the conducting wires themselves.

Weber now considers the situation where one current element follows the other along the same line, that is the situation described by Ampère's *longitudinal force* (Figure 6). If the electrostatic law alone applied, the two attractions of opposite particles (2 and 4) would exactly equal the two repulsions of like particles (1 and 3). But by the crucial *hypothesis* derived from Ampère's experiments, we know there will be an attraction or repulsion between the current elements, depending on the direction of current flow.

The question is, how must the electrostatic law be modified in order to yield the longitudinal force as a result? Notice in Figure 6, Case 1, that the particles in relative motion are those of opposite charge (the like particles flow in the same direction and thus have no relative velocity). Now see, in Case 2, that it is the like particles that are in relative motion. In Case 1, the resultant force is repulsion; in Case 2, it is attraction. From this, Weber adduces the theorem *that the electrostatic force must be reduced when the electrical particles are in relative motion,* that is when they have a relative velocity.

The electrostatic law is a simple inverse square law. If e and e' are the charge of two stationary particles, and r their distance, the force between them is simply ee'/r^2. The *relative velocity* of two particles can be designated as dr/dt. Since the theorem of Weber applies both where the particles are approaching or receding from one another—that is, where the sign (direction) of the relative velocity is either positive or negative, Weber will use the *square,* dr^2/dt^2. He thus expresses his theorem for the force between two electrical particles in longitudinal motion:

$$\frac{ee'}{r^2}\left(1 - a^2\frac{dr^2}{dt^2}\right),$$

where a is a constant whose value must be determined.

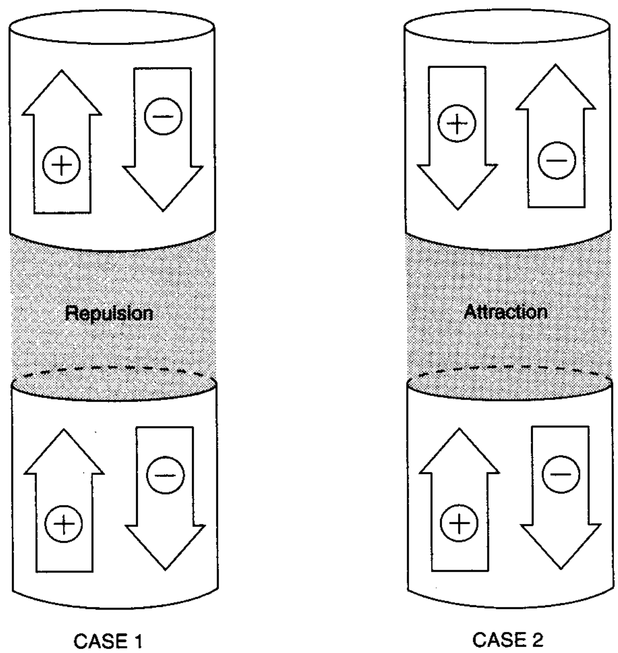


Figure 6
WEBER'S DEVELOPMENT OF AMPERE'S LAW: LONGITUDINAL ELEMENTS

To broaden Ampère's approach to include the new phenomena of induction, Weber used the hypothesis of Gustav Fechner that a current consists of the opposite flow of positive and negative electrical particles. In this view, a single current element contains a positive and a negative particle in opposite motion, depicted here by the contents of a single cylindrical section of the wire. The schematic, for each case, depicts two of these current elements, one following the other, in a straight line along the wire. In Case 1, where the current elements (positive and negative particles for Weber) are moving in the same direction, Ampère's theory deduced repulsion. For Case 2, where the positive particles and the negative particles have opposite motion, Ampère deduced attraction. From these experimental deductions of Ampère, Weber determined the velocity dependency of the law of force between electrical particles.

The Parallel Case

The same considerations must now be applied to the case of two parallel current elements which form a right angle with the line connecting their midpoints (Figure 7). In this case, the result (attraction or repulsion depending on whether the currents flow in the same or different directions), was known to Ampère through his earliest experiments. Their interaction will now be analyzed, as in the previous case, according to Fechner's hypothesis.

The first thing we notice is that at the very instant when the current elements are directly opposite each other, the *relative*

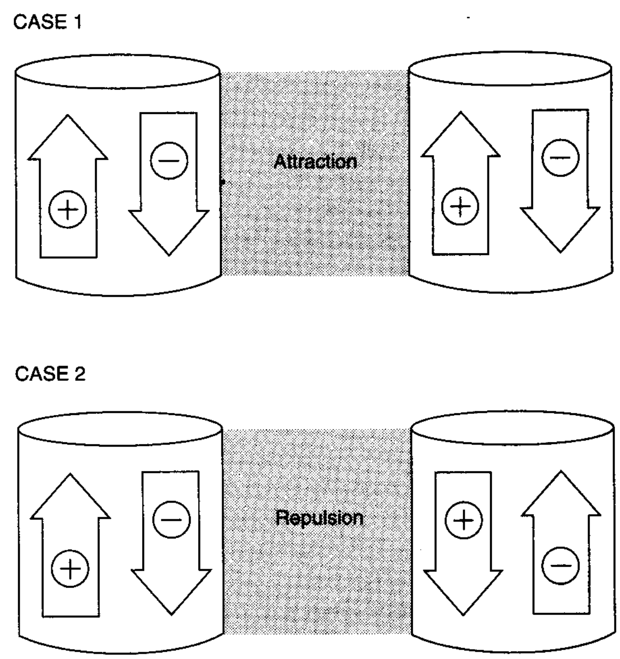


Figure 7
WEBER'S DEVELOPMENT OF AMPERE'S LAW: PARALLEL ELEMENTS

When the current elements are parallel, Ampère's theory describes attraction in Case 1, where both elements move in the same direction, and repulsion in Case 2, where the two elements move in opposite directions. Weber noted that like electrical particles move in the same direction in Case 1, and in opposite directions in Case 2. In the longitudinal case, the same relative motions of the particles produced opposite results. Weber saw that in the parallel case, the relative velocity of the particles was zero, but that they had a relative acceleration. Thus came the acceleration term in his fundamental law of electricity.

velocity of all the electrical particles is zero, and thus the law just deduced can have no bearing in explaining the resultant force. That is, two particles approaching each other, are said to have a negative relative velocity; as their paths cross, their relative velocity is zero; as they now recede from each other, their relative velocity becomes positive. At the point of crossing, the relative velocity is changing from negative to positive. A change in relative velocity is known as *relative acceleration.* Thus at the instant under consideration, when the current elements are directly opposite each other, they have a *relative acceleration,* but no *relative velocity.*

Now, in Case 1 of Figure 7, where the net effect according to Ampère's experiments is attraction, we observe that it is the *unlike* particles that are in relative motion between the two current elements. In Case 2, where the force between current elements is repulsive, we observe that it is the *like* particles which are in relative motion. Thus the situation is the opposite of that noted for longitudinal current elements, and, rather than diminishing, the electrostatic force must be increased by the presence of a relative acceleration between particles. Thus must Weber add a term to the expression derived just above, which yields:

$$\frac{ee'}{r^2}\left(1 - a^2\frac{dr^2}{dt^2} + b\frac{d^2r}{dt^2}\right), \qquad \textbf{Eq. 2}$$

where b is also a constant to be determined.

Now a more detailed consideration arises; namely, unlike the previous case, the particles in parallel current elements do not move along the same straight line. When the function of their distance of separation is determined (Weber 1846, §19), the result is that

$$\frac{d^2r}{dt^2} = \frac{1}{r}\frac{dr^2}{dt^2}.$$

Hence, Equation 2 becomes:

$$\frac{ee'}{r^2}\left(1 - a^2\frac{dr^2}{dt^2} + \frac{b}{r}\frac{dr^2}{dt^2}\right). \qquad \textbf{Eq. 2(a)}$$

To find the relationship between the coefficients a and b, Weber returns once more to Ampère's work, and specifically to the point referenced in Gauss's 19 March letter. The ratio of the coefficients in Equation 2(a) is nothing other than the ratio of the force between parallel current elements to the force between longitudinal elements; namely the same relationship which Ampère had determined to have the absolute value $1/2$. Weber consequently sets:

$$a^2 = \frac{1}{2}\frac{b}{r}, \text{ or } b = 2a^2r,$$

from which the general expression for the force between two electrical particles becomes:

$$\frac{ee'}{r^2}\left(1 - a^2\frac{dr^2}{dt^2} + 2a^2\cdot r\frac{d^2r}{dt^2}\right). \qquad \textbf{Eq. 3}$$

By consideration of the four interrelationships existing among the particles of each pair of current elements, Weber divides the constant a by 4, producing the finished 1846 form of his expression for the force between two electrical particles in motion:

$$\frac{ee'}{r^2}\left(1 - \frac{a^2}{16}\frac{dr^2}{dt^2} + \frac{a^2}{8}r\frac{d^2r}{dt^2}\right). \qquad \textbf{Eq. 4}$$

This is Weber's fundamental law of electrical action as presented in his famous 1846 memoir, in which he also shows its application to the phenomenon of induction and its complete compatibility with Ampère's law. Most of the features of atomic physics that Weber was later to discover are already implicit in the formulation stated in Equation 4.

4. The Final Steps

In 1855, Weber and Rudolf Kohlrausch carried out experiments which determined with fine precision the value of the constant so far designated as a. They found that $4/a = 4.395 \times 10^{11}$ mm/sec, and this value, thenceforth designated c, came to be known as the *Weber constant.*[2] Weber understood the constant c as "that relative velocity which electrical masses e and e' have and must retain, if they are not to act on each other at all" (Weber and Kohlrausch 1856, p. 20). His fundamental law was from now on to be written:

$$\frac{ee'}{r^2}\left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2} + \frac{2r}{c^2}\frac{ddr}{dt^2}\right). \qquad \textbf{Eq. 5}$$

In a comment appended to the précis of the experiment, published jointly with Kohlrausch in 1856, Weber hints at the direction his thought was to take in coming years. Weber pointed out that the extremely small value of the coefficient $1/c^2$

> makes it possible to grasp, why the electrodynamic effect of electrical masses . . . compared with the electrostatic . . . always seems infinitesimally small, so that in general the former only remains significant, when as in galvanic currents, the electrostatic forces completely cancel each other in virtue of the neutralization of the positive and negative electricity {Weber and Kohlrausch 1856, p. 21}.

It shall shortly become clear that Weber was already groping for a means to penetrate to the level of the forces among these tiny particles of electrical charge, those which we now call *atomic.* His comment reveals that he could not see an experimental path to that goal. The power of his subsequent work resides largely in his determined working through of the theoretical implications of his earlier work.

Catalytic Forces and a Fundamental Length

We jump now to 1870, when Weber is under a sustained attack by Helmholtz and Clausius in Germany and Thomson, Tait, and Maxwell in Britain. They are claiming that Weber's law must violate the principle of conservation of energy. Helmholtz has constructed a specific case where, he claims, Weber's law will produce an infinite *vis viva.*

In a treatise which appeared in January 1871, his sixth memoir under the series titled "Electrodynamic Determinations of

Measure" (Weber 1871), Weber not only offers a devastating reply to the criticisms, but also discovers, purely through a theoretical analysis of his fundamental electrical law, basic principles of atomic physics, which were not empirically determined until decades later. In the opening pages of the memoir is found perhaps the most astounding of these discoveries, Weber's determination of a minimal distance below which the Coulomb force, the repulsion of like particles, must reverse and become attractive.

First Weber notes that the positive and negative electrical particles, expressed as e and e', are not masses in the mechanical sense. Lacking our current use of the term, *charge,* they had been called at the time *electrical masses.* Weber draws the distinction, between charge and mechanical mass, expressing the former by e, and the latter by ϵ (epsilon). He then recognizes that while the amount of charge on positive and negative electrical particles is equal, though opposite, their masses need not be equal. He thus arrives for the first time, on page 3 of the *Sixth Memoir,* at the modern concepts of charge-to-mass ratio and proton-electron mass ratio.

Chris Lewis

Jonathan Tennenbaum observing the ultra-sensitive receiving apparatus of the Gauss-Weber telegraph, set up here to detect very weak magnetic forces. The telegraph, the world's first, was constructed in 1833, and ran from the Göttingen Observatory to the Physics Building.

Weber next examines an underlying assumption in his fundamental law of electrical action. Namely, that the expression for the force, which the particles, e and e', mutually exert upon each other, is dependent on a magnitude, that is, their relative acceleration, "which contains as a factor the very force that is to be determined." He makes this clear by the consideration that the relative acceleration must consist of two parts—one due to the mutual action of the two particles, and a second part due to other causes. The second part would include whatever velocity the particles may have in directions other than the line r connecting them, and whatever is due to the action on them by other bodies. He had already considered this aspect of the matter in the 1846 memoir (p. 212 ff.), where he employs the term *catalytic forces* to describe them, after the expression introduced by the chemist Berzelius. In that location, by considering separately the mathematical term for the force of acceleration which each one of the particles exerts on the other one, he was able to derive an expression for his fundamental law which is independent of the acceleration term caused by their mutual action, but which still must contain a term, f, which denotes the acceleration due to other causes. The expression thus derived is

$$\frac{ee'}{rr - \frac{2r}{cc}(e + e')}\left(1 - \frac{1}{cc}\frac{dr^2}{dt^2} + \frac{2rf}{cc}\right). \quad \textbf{Eq. 6(a)}$$

But when the distinction is made between the charge (e,e') and the mass (ϵ, ϵ') of the electrical particles, Weber shows in the *Sixth Memoir* (Weber 1871, pp. 2-6) that the expression then becomes:

$$\frac{ee'}{rr - \frac{2r}{cc}\frac{\epsilon + \epsilon'}{\epsilon\epsilon'}ee'}\left(1 - \frac{1}{cc}\frac{dr^2}{dt^2} + \frac{2rf}{cc}\right). \quad \textbf{Eq. 6(b)}$$

"From this it results," Weber remarks, "that the law of electrical force is by no means so simple as we expect a fundamental law to be; on the contrary, it appears in two respects to be particularly complex." The first complexity is the *catalytic forces* just referenced. The second is the appearance of a unique length, associated with reversal of the Coulomb force. As Weber describes that latter aspect of the discovery:

> In the second place, another noteworthy result follows from this expression for the force—namely, that when the particles e and e' are of the same kind, *they do not by any means always repel each other;* thus when $dr^2/dt^2 < cc + 2rf$, they repel only so long as
>
> $$r > \frac{2}{cc}\frac{\epsilon + \epsilon'}{\epsilon\epsilon'}ee' ,$$
>
> and, on the contrary, they attract when
>
> $$r < \frac{2}{cc}\frac{\epsilon + \epsilon'}{\epsilon\epsilon'}ee' .$$

This remarkable result is no more than a necessary, mathematical consequence of the expression for Weber's fundamental law just given above. It is easily seen that when e and e' represent two similar particles, the expression gives $4e^2/\epsilon c^2$. Recalling that the Weber constant is $\sqrt{2}\times$ the velocity of light, we have then in modern terms, where c expresses the velocity of light, and m_e the mass of the electron, the familiar expression

Critics of Ampère and Weber

Ampère's revolutionary hypothesis that magnetism arises from electrical orbits surrounding the particles of matter became the basis for the development of early atomic science by Gauss, Weber, and others. But many could not understand his hypothesis, nor deal with the fact that its mathematical development implied an overthrow of Newtonian mechanics.

Among the leading critics of Ampère and Weber's work were Hermann Grassmann, James C. Maxwell, and the English engineer Oliver Heaviside. Grassmann attacked the Ampère hypothesis as "improbable," but without giving a reason. Maxwell took the middle ground of allowing Ampère's hypothesis, but rejecting Weber's development of it. Heaviside's position is notable as an expression of the sort of gross empiricism so frequently encountered in science today. His suggestion, that Ampère's contribution be changed to what it was not, has been adopted by most modern textbooks.

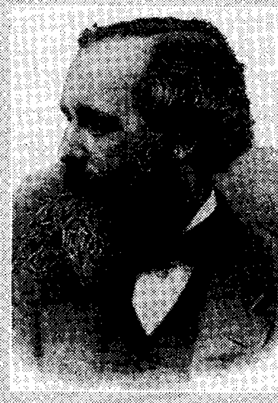
Hermann Grassmann
(1809-1877)

When I submitted the explanation offered by Ampère for the interaction of two infinitely small current-sections on one another to a more exacting analysis, this explanation seemed to me a highly improbable one. . . .

—Hermann Grassmann,
A New Theory of Electrodynamics, 1845

There are also objections to making any ultimate forces in nature depend on the velocity of the bodies between which they act. If the forces in nature are to be reduced to forces acting between particles, the principle of the Conservation of Force requires that these forces should be in the line joining the particles and functions of the distance only.

—James Clerk Maxwell,
On Faraday's Lines of Force,
1854

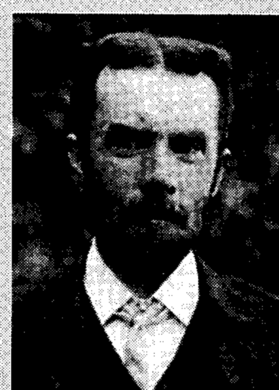


Millikan and Gale, A First Course in Physics (Boston: Ginn & Co. 1915)

James Clerk Maxwell
(1831-1879)

It has been stated, on no less authority than that of the great Maxwell, that Ampère's law of force between a pair of current elements is the cardinal formula of electrodynamics. If so, should we not be always using it? Do we ever use it? Did Maxwell in his Treatise*? Surely there is some mistake. I do not in the least mean to rob Ampère of the credit of being the father of electrodynamics: I would only transfer the name of cardinal formula to another due to him, expressing the mechanical force on an element of a conductor supporting current in any magnetic field—the vector product of current and induction. There is something real about it; it is not like his force between a pair of unclosed elements; it is fundamental; and, as everybody knows, it is in continual use, either actually or virtually (through electromotive force), both by theorists and practicians.*

—Oliver Heaviside,
in *The Electrician*, 1888



IEE Archives

Oliver Heaviside
(1850-1925)

$$\frac{2e^2}{m_e c^2} .$$

That is the distance below which two electrons may not approach, which, when divided by two, gives the classical electron radius of 2.8×10^{-13} cm. When the mass of the proton is inserted into the Weber expression, the value 3.06×10^{-16} cm results—perhaps some sort of lower bounding value for the strong force, in any case, a most interesting approximation for the year 1870!

Weber's 1871 paper progresses in richness. The laws of motion of an electron orbiting around a central nucleus are deduced, and the determination that two like particles cannot have such orbital motion, but that an oscillation along the same line (as if attached by a rubber band) is possible. (From this latter, Weber attempts to find the basis for the production of oscillations of the frequency of light.) Finally, Helmholtz's silly charge that the electrical particles will attain an *infinite energy* under Weber's formulation, is answered with the observation that Helmholtz assumes the possibility of the particles also attaining *infinite relative velocities*. Rather it is the case, Weber points out, that his constant, *c*, must represent a *limiting velocity* for the electrical particles.

These are some of the remarkable results produced by that singular intervention of Gauss in his letter of 19 March 1845. His recognition of the *real existence* of a mere *idea*, which had appeared in the mind of Ampère by no later than 1823, led to the discovery of some of the most crucial among the concepts of our modern physics. Thus did a scientific *idea*, a colorless, odorless, and tasteless substance, change the course of history.

Yet, none of these remarkable accomplishments of the line of work from Ampère to Gauss to Weber, which we have just reviewed, is recognized in the standard histories, or textbooks today. Only when one delves into the remote corners of specialist sources can scant mention of these facts be found—always presented as isolated events, never with coherence. How can it be that the names of Ampère, Gauss, and Weber are never mentioned when the physics of the atom is taught? If the name of Ampère arises, it is in connection only with electricity and magnetism. The actual law of electrical force he discovered is almost impossible to find in any modern textbook; under Ampère's name appears something quite different. The name of Weber is rarely heard.

Today, students of physics and electrical engineering are taught that all of the laws of electrodynamics have been included under the ingenious formulations arrived at by James Clerk Maxwell and codified in his 1873 *Treatise on Electricity and Magnetism*. One need not study Ampère and Weber, they are told, because Maxwell already did that. He also did us the service of cleaning up any "errors" that might have been found there. And a very thorough job it was.[3]

But where, pray tell, did the method of discovery go? Or is that no longer of interest to students today? Is it that we know so much today, that it would only be confusing to teach how we know it? (Some might even be so foolish as to argue thusly.) Yet not just the method is missing. So too are its results. Where did the classical electron radius, the nuclear strong force, the limiting value of the velocity of light come from? Not from Ampère, Gauss, and Weber, according to today's textbooks and authorities. Did we in any way exaggerate when we used the term *a consumer fraud* to describe the university science education which commits such glaring omissions? Has a fraud been committed, a cover-up? Was it accidental or witting? We hope we have given the reader sufficient leads that he may investigate and decide for himself.

Laurence Hecht is an associate editor of 21st Century. *A co-thinker of Lyndon H. LaRouche, he is currently a political prisoner in the state of Virginia.*

Notes

1. The mathematical development of Ampère's hypothesis, of a force acting along the straight line conecting two elements, and certain uncritical references to Newton found in the opening pages of his 1826 Memoir, have emboldened some interpreters, Maxwell included, to falsely presume Ampère to be a Newtonian. They completely miss the point. Ampère's 1826 Memoir is rather a sort of Gödel's proof for experimental physics: working within the framework of Newtonian assumptions to demonstrate the absurdity of sticking to the Newtonian assumptions of point mass and a simply continuous, linear-extended space-time. Without referencing it explicitly, Ampère is raising precisely the same points of criticism of Newtonian assumptions addressed a century earlier by Gottfried W. Leibniz in his famous correspondence with Newton's proxy Samuel Clarke, and in his *Monadology.* Immediately following the completion of his 1826 Memoir on electrodynamics, Ampère turned his attention to these deeper implied issues of his experimental work, becoming a champion of Leibniz's method in science from that time until his death in 1836.

   The continuing hegemony within mathematical physics, even to the present moment, of Leonhard Euler's fraud respecting Leibniz's work, is the root of the failure of Maxwell and all subsequent specialists to recognize this essential aspect of Ampère's contribution. Where science must answer such questions by experimental measurement, Euler claims to refute Leibniz's insistence on the existence of atomic structure within the "hard, massy partickles" of Newton's cosmology, by resorting in his *Letters to a German Princess,* to a blackboard trick. To defend the existence of a mere mathematical construct, his ever-present infinite series, Euler claims to prove the *physical existence* of a simply continuous space-time by successively subdividing a straight line into as many parts as the mind can imagine ("as near as you please" in Augustin Cauchy's more refined version of the ruse). Thus is reality stood on its head by a mathematician's trick, passed on from generation to credulous generation of university science undergraduates.

   See also, Lyndon H. LaRouche, Jr, "Riemann Refutes Euler," *21st Century Science & Technology,* (Winter 1995-1996) pp. 36-47.

2. The experiment actually determined the ratio of the mechanical measure of current intensity to the three other existing measures, that is, the electromagnetic, the electrodynamic, and the electrolytic. The value given above, the *Weber constant,* is the ratio of the mechanical to the electrodynamic measure. Weber first showed in 1846 that the ratio of the electrodynamic to the electromagnetic unit is as $\sqrt{2}:1$. Therefore, the experimentally derived ratio of the mechanical measure of current intensity to the electromagnetic measure was 3.1074×10^{11} mm/sec. Bernhard Riemann, who observed the Weber-Kohlrausch experiment, was the first to note that the value corresponded closely to Fizeau's experimental determination of the velocity of light. His theory of *retarded potential* proceeded from there. Here Riemann attempted the unsolved task of which Gauss had commented in the 19 March 1845 letter:

   > Without a doubt, I would have made my investigations public long ago, had it not been the case that at the point where I broke off, what I considered to be the actual keystone was lacking . . . namely, the *derivation* of the additional forces (which enter into the reciprocal action of electrical particles at rest, if they are in relative motion) from the action which is *not instantaneous,* but on the contrary (in a way comparable to light) propagates itself in time.

3. Maxwell was a capable mathematical analyst and possessed a creative gift for physical-geometrical insight. His utter ignorance of matters of method, which took the form of a slavish adherence to the method of empiricism, prevented his ever understanding the deeper issues posed by Gauss above (note 2). Maxwell stubbornly remarks on Gauss's challenge:

   > Now we are unable to conceive of a propagation in time, except either as the flight of a material substance through space, or as the propagation of a condition of motion or stress in a medium already existing in space [*Treatise,* p. 492].

   Maxwell's dismissal of what he did not understand, increasingly took on the character of ignorant prejudice. There was nothing original in his idea of an ether as the transmitting medium of electromagnetic action. Had Gauss seen a clear solution through such a mode of representation, he would have developed it. There was none, as the glaring failure of Maxwell's theory to even account for the existence of the electron ought to indicate.

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Experimental Apparatus and Instrumentation

1. The Gauss Magnetometer

Carl Friedrich Gauss's 1832 determination of the absolute intensity of the Earth's magnetic force was the crucial prerequisite for Weber's electrodynamic studies. Prior to Gauss's work, measurements of the intensity of the Earth's magnetic force were carried out by observing the oscillations of compass needles at varying points on the Earth's surface. Based on the theory of the pendulum, the intensity was assumed to be equal to the square of the number of oscillations.

Observations of both the horizontal and vertical (inclination and dip) intensity had been carried out sporadically over the previous century and brought to a high state of refinement by the travels of Alexander von Humboldt. But all of these observations contained an inherent weakness: that the magnetic strength of the needles used had to be considered equal and unchanging. A more exact determination of the magnetic intensity at differing points on the Earth's surface had long been desired for a better understanding of geomagnetism, which would be useful in navigation, surveying, and the Earth sciences. It was soon to play a crucial role as well in the theoretical pursuit of electrodynamics and atomic theory.

The problem in all the observations carried out prior to Gauss's work, was that the strength, or *magnetic moment, M,* of the oscillating needle could not be separated from the strength of the Earth's magnetism, *T.* The number of oscillations observed is proportional to the product of the two, *MT.* Thus, it is impossible to tell whether variations measured at different points on the Earth's surface, or at different times in the same location, represent changes in the intensity of the Earth's magnetism, or are a result of a natural weakening of the needle's magnetism.

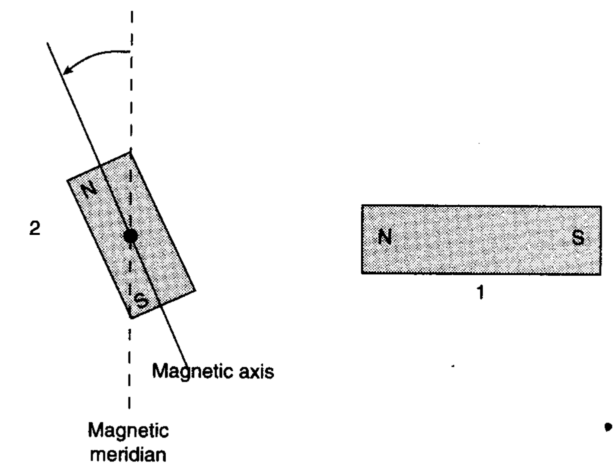


Figure 1.2
GAUSS'S METHOD
Observations carried out by Poisson's method either totally miscarried, or produced very approximate results. In this configuration, conceived by Gauss, needle 1 tends to produce an angular deflection in the second, oscillating needle, while the Earth's magnetism attempts to return the second needle into a line with the magnetic meridian. The resulting angular deflection is proportional to the sought-for ratio M/T. An additional apparatus devised by Gauss, known as the Spiegel und Fernrohr (mirror and telescope), allowed for the precise determination of the angular deflection to a hitherto unknown degree of accuracy. (See Figure 1.3).

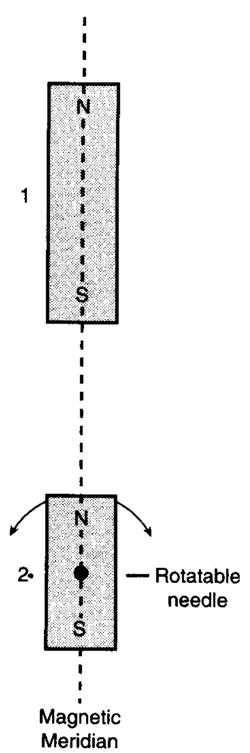


Figure 1.1
POISSON'S METHOD

The oscillations of the needle marked 2 *will be accelerated by the presence of the fixed, first needle in this configuration, where the opposite poles are turned toward each other. A comparison of the number of oscillations in this configuration, to the number of oscillations when the first needle is removed, gives the ratio* (M/T) of the magnetic strength of the first needle to the Earth's magnetic strength.

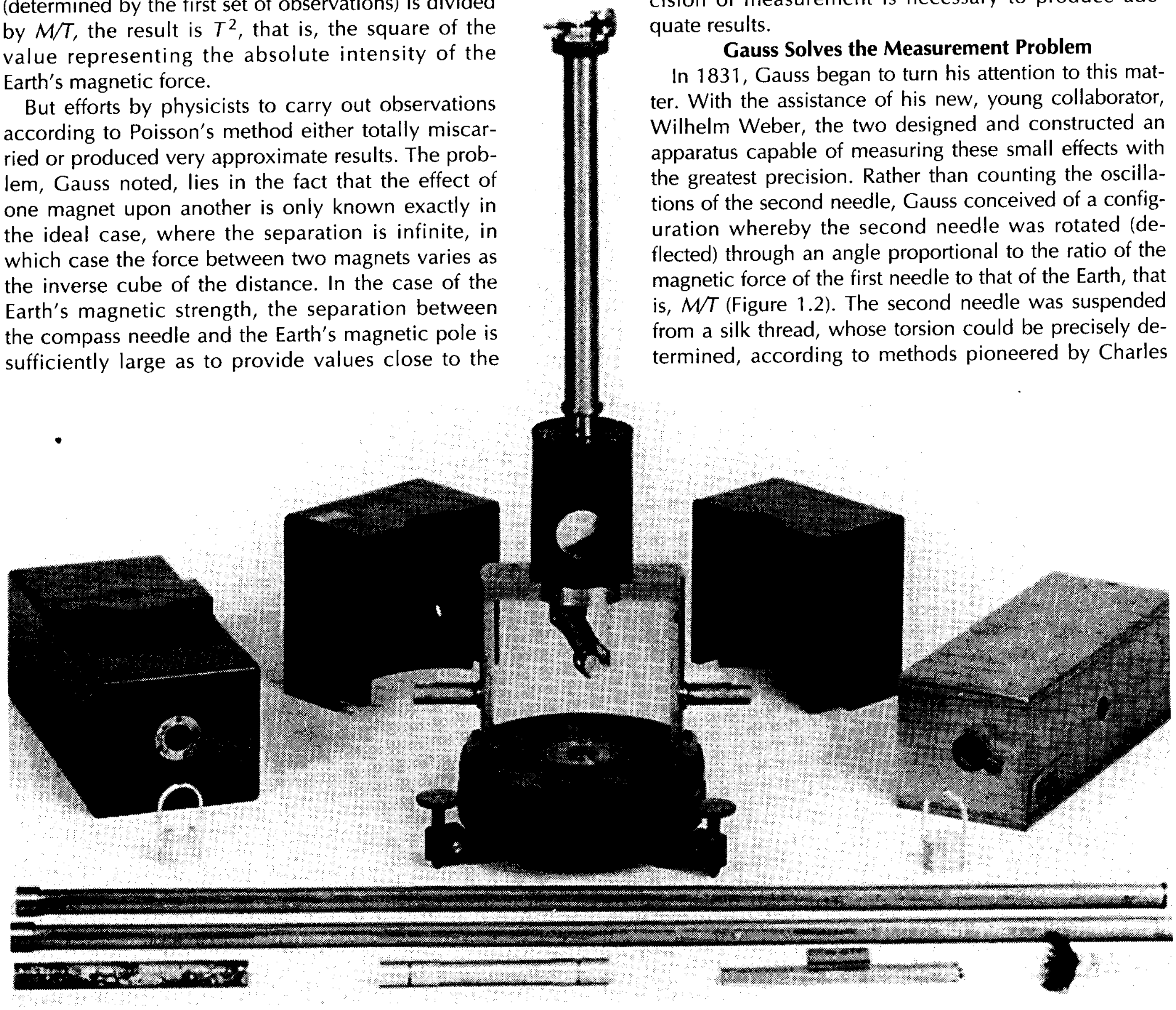
Before Gauss, Poisson in France had suggested a means of overcoming this obstacle, by making a second set of observations on the compass needle whose oscillations, under the influence of the Earth's magnetic force, had already been observed. Poisson proposed to fix this needle in line with the magnetic meridian (that is, pointing to magnetic north). A second, rotatable or oscillating needle was then to be placed in the same line (Figure 1.1).

The oscillations of the second needle would be either retarded or accelerated by the presence of the first needle, according to whether like or unlike poles are turned toward each other. By comparing the number of oscillations of the second needle when in the presence of the first one, to its oscillations when standing alone (that is, under the sole influence of the Earth's magnetic force), the ratio *M/T*, expressing the strength of the first needle to the magnetic strength of the Earth, could be determined. When the value of *MT* (determined by the first set of observations) is divided by *M/T*, the result is T^2, that is, the square of the value representing the absolute intensity of the Earth's magnetic force.

But efforts by physicists to carry out observations according to Poisson's method either totally miscarried or produced very approximate results. The problem, Gauss noted, lies in the fact that the effect of one magnet upon another is only known exactly in the ideal case, where the separation is infinite, in which case the force between two magnets varies as the inverse cube of the distance. In the case of the Earth's magnetic strength, the separation between the compass needle and the Earth's magnetic pole is sufficiently large as to provide values close to the ideal case. However, to carry out the second observation on two compass needles of finite separation—as suggested by Poisson—with any degree of accuracy, requires that the separation distance of the two needles be rather large in relation to the length of the second one. This means that the observable effect is very small, and therefore the greatest precision of measurement is necessary to produce adequate results.

Gauss Solves the Measurement Problem

In 1831, Gauss began to turn his attention to this matter. With the assistance of his new, young collaborator, Wilhelm Weber, the two designed and constructed an apparatus capable of measuring these small effects with the greatest precision. Rather than counting the oscillations of the second needle, Gauss conceived of a configuration whereby the second needle was rotated (deflected) through an angle proportional to the ratio of the magnetic force of the first needle to that of the Earth, that is, *M/T* (Figure 1.2). The second needle was suspended from a silk thread, whose torsion could be precisely determined, according to methods pioneered by Charles



Historical Collection of the Göttingen University I. Physical Institute

A transportable magnetometer built for Wilhelm Weber in 1839 by Meyerstein. The apparatus in the center is used to determine the absolute intensity of the Earth's magnetic force. In the first row, in foreground, are two bar magnets with cleaning brush and holder. Behind it are two brass bars which can be attached to the magnetometer housing at either of the two flanges seen protruding to the left and right. The bar magnet, which plays the role of needle 1 in the schematic of Figure 1.3, is slid along this non-magnetic brass support until the proper distance is achieved.

Suspended in the center of the magnetometer housing is a rotatable carrier holding the cylindrical magnetized needle, which plays the role of needle 2 in Figure 1.3. The rotatable carrier is suspended by two silk threads which run up the vertical column to the highest point of the apparatus (44 cm). Attached to the carrier is a plane mirror, which is observed through the porthole in the dark cylindrical casing above the rotatable magnet. A telescope and meter stick such as that pictured on page 23 would be aimed at the mirror.

The boxes, at far left and right, hold weights used to determine the gravitational moment of the magnet. In the background are a wooden housing to protect the apparatus from air currents and a copper one for damping the oscillations of the needle with electrical current.

Augustin Coulomb in France, several decades earlier. Then, through an ingenious apparatus conceived by Gauss, the angle of deflection could be measured with a degree of precision hitherto unknown.

Gauss's angle measurement apparatus, the *Spiegel und Fernrohr* (mirror and telescope), was integrated into many types of precision measuring instruments well into this century. Figure 1.3 schematically portrays one of the earliest versions of the apparatus. In later versions, the mirror was attached at the rotational axis of a carrier holding the second compass needle, or magnet (see photograph of 1839 device, page 36). Another important breakthrough, also incorporated into the pictured 1839 device, was the development of the *bifilar* (two-thread) suspension. By varying the distance of separation of the two parallel silk threads supporting the rotatable compass needle, their torsion could be adjusted with the greatest precision. Since the torsion provided a part of the restoring force, against which the angular rotation of the second needle by the first had to operate, its exact determination was essential for experimental accuracy.

Gauss's 1831-1832 study of magnetism, reported in his paper "The Intensity of the Earth's Magnetic Force Reduced to Absolute Measure,"[1] became the model for all rigorous investigation thereafter. The study included the first introduction of the concept that the units of *mass, length,* and *time* could serve as the basis for all physical measurement.

Notes

1. "Intensitas vis magneticae terrestris ad mensuram absolutam revocata," read by Gauss at the Göttingen Gesellschaft der Wissenschaften on 15 December 1832, and printed in Volume 8 of the treatises of this society, pp. 3-44.

   German translation from the orginal Latin, by Dr. Kiel of Bonn, available as: *Die Intensität der Erdmagnetischen Kraft auf absolutes Maass zurückgeführt* (Leipzig: Wilhelm Engelmann Verlag, 1894).

   English translation (unpublished) from the German, by Susan P. Johnson.

Chris Lewis

The boxed portable magnetometer, on display in the Historical Collection of the Göttingen University I. Physical Institute.

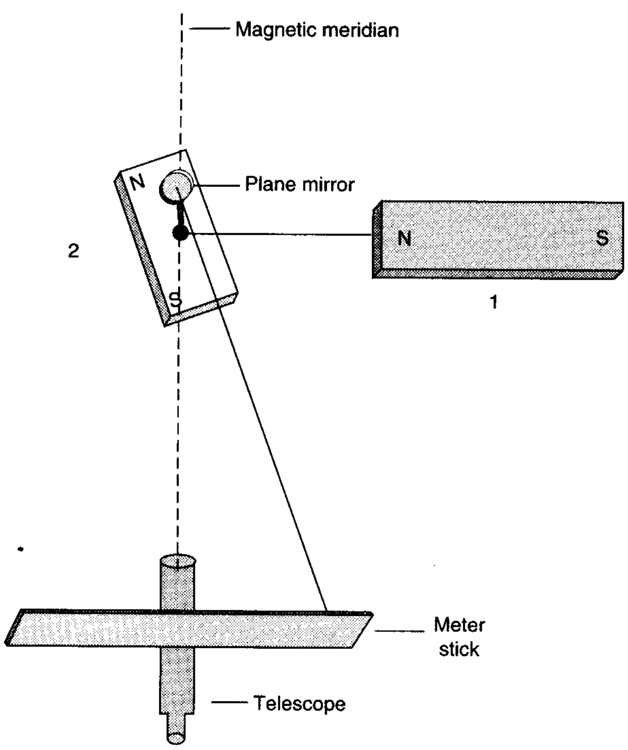


Figure 1.3
***SPIEGEL UND FERNROHR* APPARATUS**

The Spiegel und Fernrohr *(mirror and telescope) apparatus, devised by Gauss, permits the very precise determination of angular deflection. A plane mirror is attached to the axis of the rotatable compass needle (2). The face of the mirror is oriented perpendicular to the magnetic axis.*

A telescope is placed about 2 meters distant, its optical axis in a line with the magnetic meridian. A graduated meter stick is affixed to the telescope, perpendicular to its optical axis. In the rest position, when the axis of needle 2 is aligned with the magnetic meridian, the mirror reflects the mid- or zero-point of the meter stick into the barrel of the telescope, and to the viewer. When the presence of needle 1 causes an angular deflection of the second needle (pictured), the resulting rotation of the plane mirror causes it to reflect a more distant part of the meter stick into the telescope. In the 1841 apparatus used by Weber, each graduated marking on the meter stick represented about 18 seconds of arc, or 1/200 of a degree.

2. The Electrodynamometer and Weber's Proof of Ampère's Theory

Commenting in 1846, on the state of electrical science since the 1826 publication of Ampère's famous memoir on electrodynamics, Wilhelm Weber wrote:

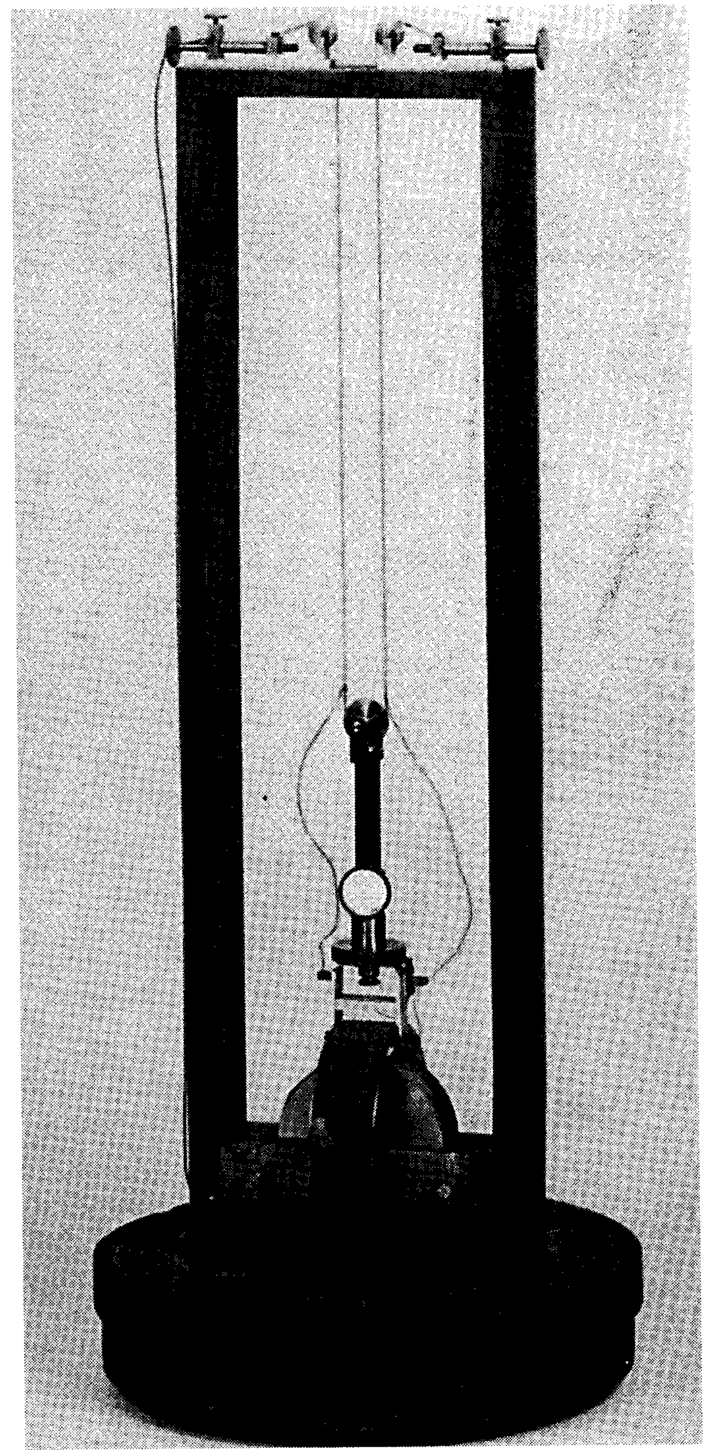
> Ampère did not continue these investigations, nor has anyone else published anything to date, from either the experimental or theoretical side, concerning further investigations. . . . This neglect of electrodynamics since Ampère is not to be considered a consequence of attributing less importance to the fundamental phenomenon discovered by Ampère . . . but rather it results from dread of the great difficulty of the experiments, which are very hard to carry out with present equipment. . . .[Weber 1846, Introduction]

The essential problem Weber saw with Ampère's apparatus was the possibility that the force of *friction* might be disguising subtle effects. In each of Ampère's equilibrium experiments, deductions are made from the lack of motion of a movable conductor, for example, the rectangular conductor CDGH in the second equilibrium experiment (Figure 4 in article text). If this lack of motion were the result, even in small part, of frictional resistance, then the entire set of deductions derived by Ampère would have to be re-evaluated.

To establish the validity of Ampère's theory with more exactness, it was necessary to devise an apparatus in which the electrodynamic forces were strengthened, such that friction would be only a negligible fraction of the force measured. This was the purpose of the instrument, known as the electrodynamometer, the first model of which Weber constructed in 1834.

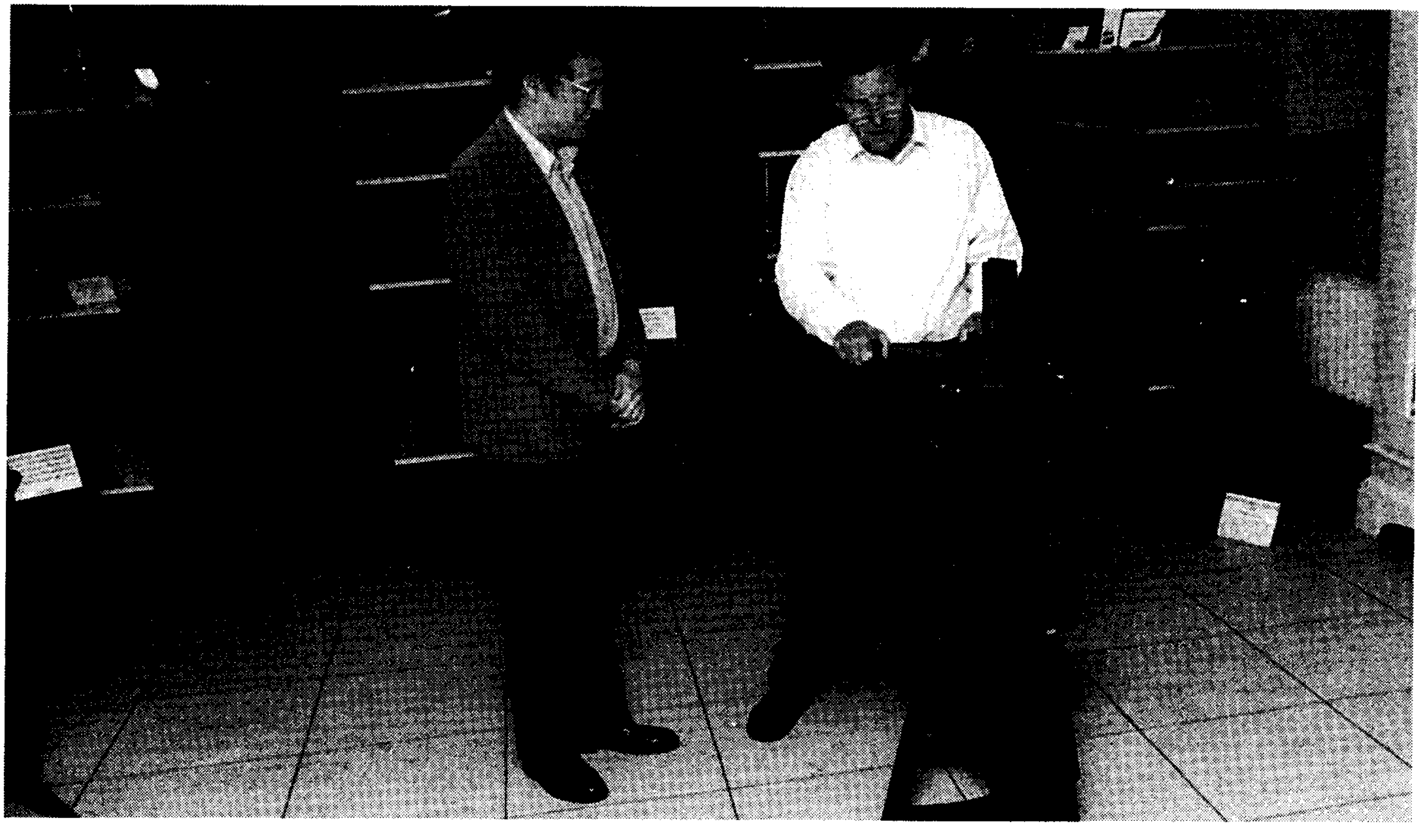
The essential improvement over Ampère's various apparatuses was, that instead of single wires interacting with each other, a pair of multiply wound coils was used. This had the advantage that each successive winding would multiply the effect of the electrodynamic force between the two coils. Thus, even the smallest currents flowing through the coils could produce measurable effects. But the use of coils, rather than single lengths of wires, would require a completely different experimental geometry. And, rather than attempting experiments whose purpose was to produce zero motion, Weber intended to precisely measure the rotational force exerted by one coil on another. Then, by geometric analysis, he would reduce these results to the effect of a single circular loop on another, and, through further analysis, relate the strength of this effect to that predicted by Ampère's law.

The principal elements of Weber's apparatus were two cylindrical coils of wire, called solenoids by Ampère. One cylinder was suspended horizontally such that it could rotate around a vertical axis. The other was placed horizontally in a fixed position, usually either perpendicular (Figure 2.1) or longitudinal to the first coil. We know from Ampère's earliest experiments, that when current passes through a solenoid, it takes on the properties of a bar magnet, one end of the cylinder acting as north pole, and the other as south. Thus, as can be seen from Figure 2.1, the arrangement of the Weber electrodynamometer



Historical Collection of Göttingen University I. Physical Institute

This is the electrodynamometer, constructed in 1841, which Weber used to experimentally verify Ampère's electrodynamic theory. The larger outer ring is the bifilar coil, *so called because it is suspended from above by two wires, which also carry current to it. The inner ring, an electrical coil known as the* multiplier, *is affixed to a wooden frame with tripod base. During the experiment, the multiplier and frame are placed in various positions on the laboratory table to determine its rotational effect on the bifilar coil. The angle of rotation is measured by observing the mirror (affixed to the suspension apparatus) through a telescope with meter stick attached.*



Chris Lewis

Professor G. Beuermann (r.) of Göttingen University demonstrates the sending apparatus of the 1833 electromagnetic telegraph of Gauss and Weber to Jonathan Tennenbaum. In the background is displayed part of the historical collection of Weber's apparatus.

is quite analogous to that of the Gauss magnetometer.

Weber borrowed the use of the bifilar (two-thread) suspension from this earlier instrument, but instead of silk threads, he used the conducting wires themselves to suspend the coil. Thus, a hollow wooden cylinder wound with insulated copper wire, which came to be known as the *bifilar coil,* was suspended from above by its own two wire leads. The second cylindrical coil, known as the *multiplier,* was placed in the same horizontal plane, at right angles, or longitudinal to the first. A mirror was affixed to the bifilar coil, and its angle of rotation observed with a telescope and meter stick, just as in the magnetometer.

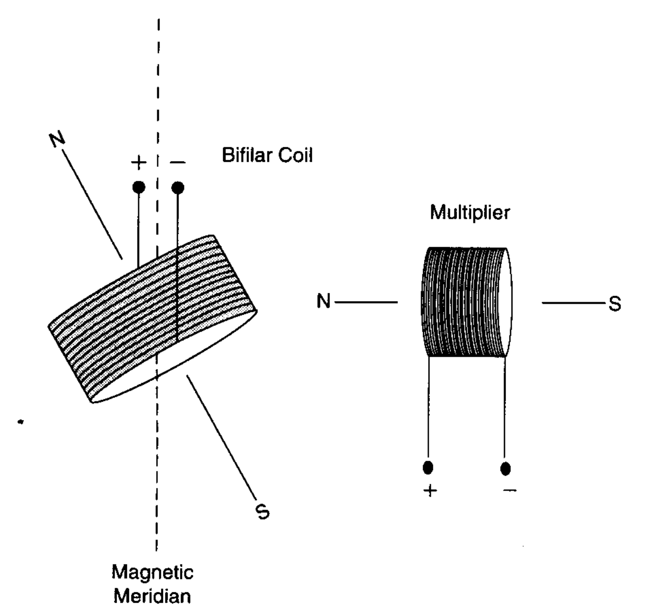


Figure 2.1
BASIC CONFIGURATION OF ELECTRODYNAMOMETER

In this topdown view of the Weber electrodynamometer, the rotatable bifilar coil is suspended by its two conducting wires. When current is passed through it, it will tend to behave just like a magnetic compass needle, aligning itself with the magnetic meridian. But the multiplier, placed at right angles to the bifilar coil, will also behave like a magnet when electrified, and will tend to rotate the bifilar coil out of the magnetic meridian, as depicted. If a plane mirror is attached to the bifilar coil and observed through a telescope and measuring stick apparatus, as in the Gauss magnetometer, the angle of rotation can be very precisely measured.

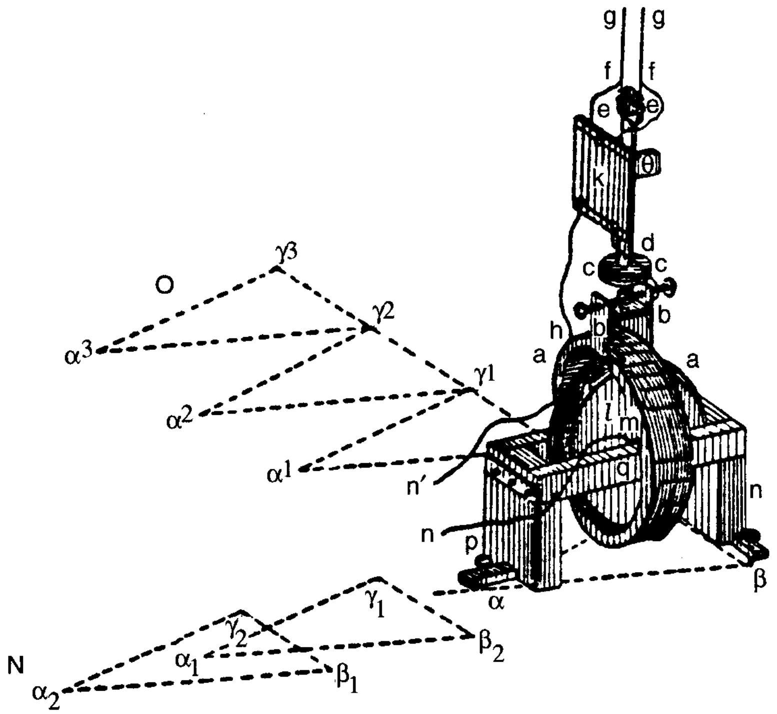


Figure 2.2
BIFILAR COIL AND MULTIPLIER
In this illustration from Weber's First Memoir, the multiplier coil is depicted in the unique position where it is inside *the bifilar coil. The bifilar coil is the outer ring, shown with the suspension apparatus leading up to the wire leads gg. A plane mirror, k, is attached to the suspension. The multiplier coil is affixed to the wooden base which is mounted on the three feet, α, β, and γ. The multiplier and base can be extracted from the position depicted and moved to any desired position. The dotted triangles indicate the premeasured positions on the laboratory table at which the multiplier will be placed during the experiment.*

After additional instrumentation was added to measure the precise current flow through each coil, observations were made with the multiplier positioned at varying, precisely measured distances to the east, west, north, and south of the bifilar coil (Figures 2.2, 2.3). A table of experimentally determined values was then arrived at, representing the torque, or rotational moment, exerted by the multiplier on the bifilar coil at the different distances. By knowing the number of turns in each coil, and by assuming from the symmetry of the windings, that the total effect could be considered as concentrated in the most central loop of each coil, Weber was then able to reduce these observed values to the mutual effect of a single pair of circular loops, acting at each measured position of the multiplier and bifilar coil.

In his mathematical theory of electrodynamics, Ampère had developed a formula that provided a theoretical determination of what the rotational moment of two such circular loops should be, dependent on their distance of separation, the area enclosed by each, their relative angles, and the strength of current flowing in them. Weber was now able to compare the predicted values, derived from Ampère's controversial theory of electrodynamics, to a set of experimentally determined values. The difference amounted to less than 1/3 of a scale unit (about 6 seconds of arc), of which Weber wrote in his First Memoir:

> This complete agreement between the values calculated according to Ampère's formula and the observed values (namely, the differences never exceed the possible amount contributed by unavoidable observational error) is, under such diverse conditions, a full proof of the truth of Ampère's law [Weber 1846, §8].

—Laurence Hecht

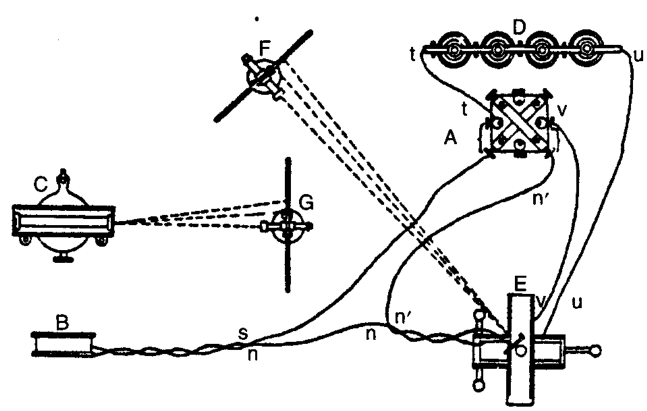
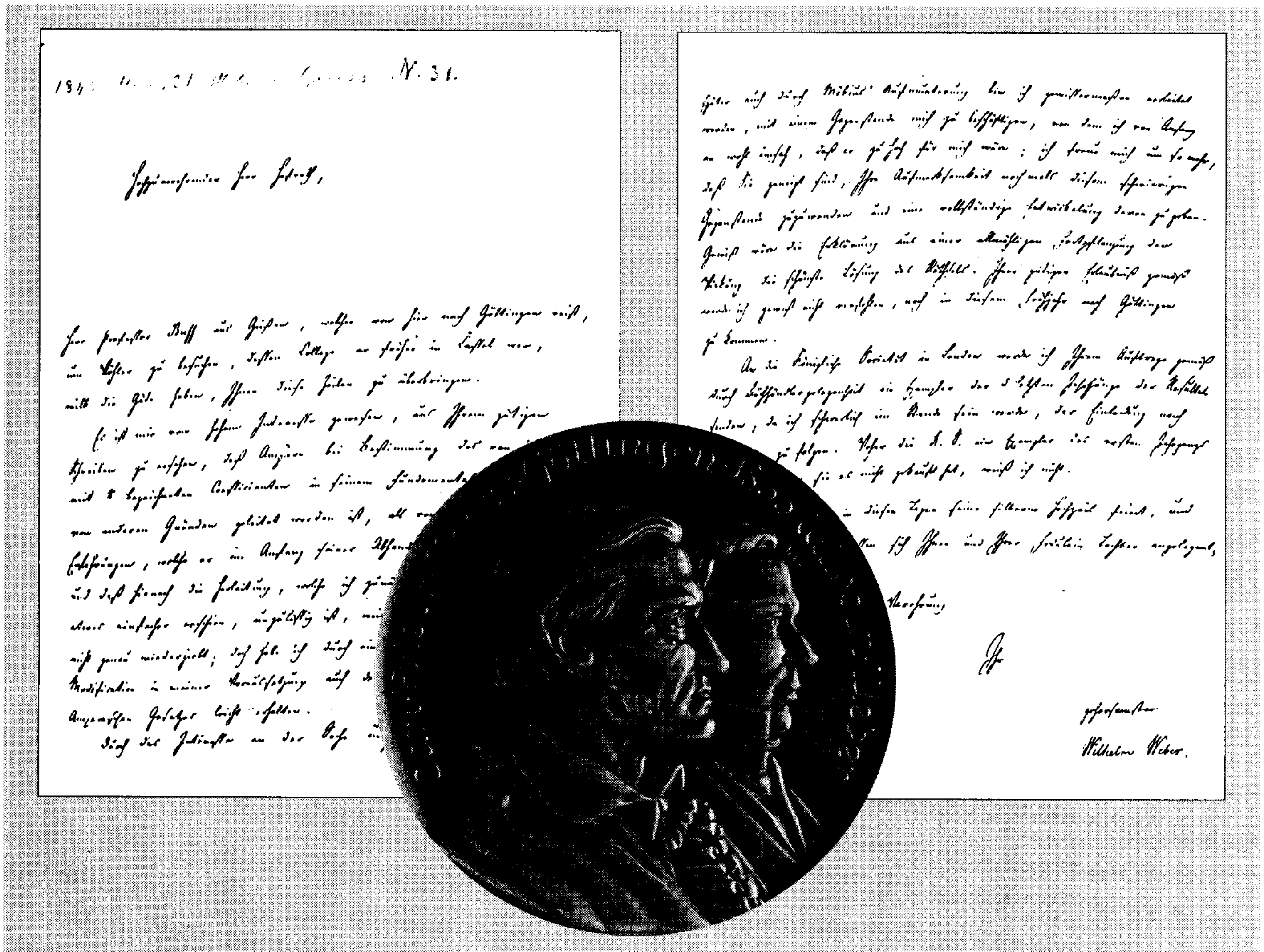


Figure 2.3
SCHEMATIC OF WEBER'S EXPERIMENT
In addition to the bifilar coil and multiplier, depicted in the closed position at E, this schematic diagram from Weber's First Memoir shows the other instrumentation required for the verification of Ampère's electrodynamic theory.

The telescope and meter stick for observing the rotation of the bifilar coil is shown at F. The current supply (a four-cell battery) is depicted at D and a commutator for reversing the direction of current flow at A. The apparatus at B, C, and G measures the current in the circuit and takes the place of a modern ammeter. B is a second multiplier coil connected to the main circuit, and about 20 feet distant from the bifilar coil. C is a portable magnetometer whose deflections (measured by the telescope and meter stick at G) correspond to the current strength in B. Observers were required at both the scopes F and G, to take simultaneous readings of the deflection of the bifilar coil and the current strength, and a third operator to manipulate the current supply.



Courtesy of the George August Physics Institute at Göttingen University

Text of the Gauss-Weber 1845 Correspondence

EDITOR'S NOTE

The letters from Weber to Gauss, numbered 29 to 31, come from the Gauss manuscripts in the Manuscripts and Rare Books Division of the State and University Library of Lower Saxony, in Göttingen. They were transcribed from the German script by Karl Krause and Alexander Hartmann. The letter from Gauss to Weber of 19 March appears in Carl Friedrich Gauss, Werke, *Vol. V, pages 627-629. All the letters were translated into English by Susan P. Johnson. The words in brackets are added by the translator; the footnotes are by the editor.*

Above: Commemorative medal honoring Carl Friedrich Gauss and Wilhelm Weber, issued in 1933. In background is a facsimile of Weber's 31 May 1845 letter to Gauss.

Weber to Gauss,
No. 29, 18 January 1845

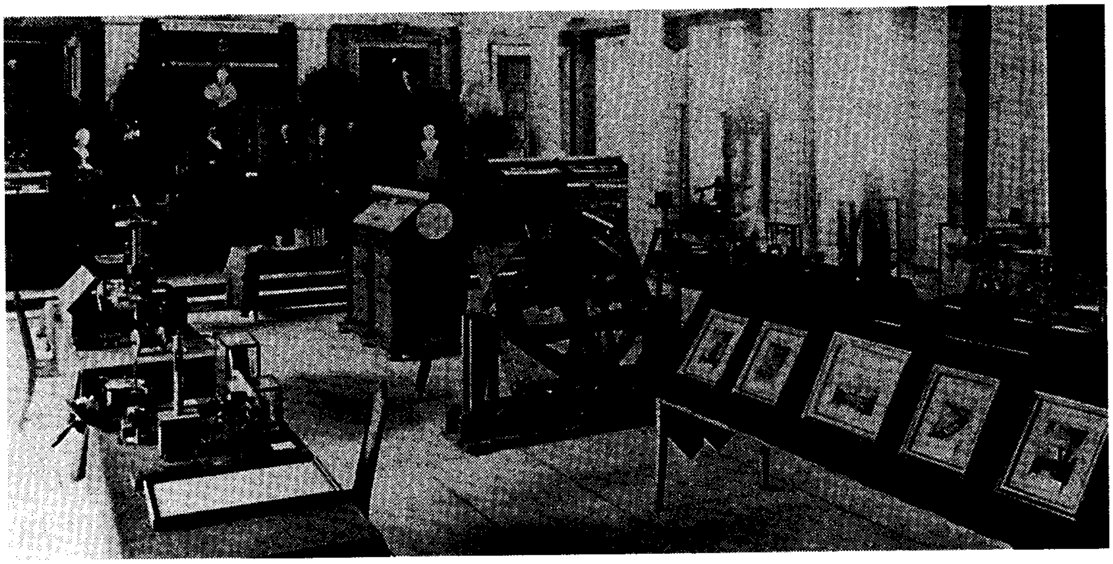
Highly honored Herr Hofrath:[1]

. . . For some time now, I have occupied myself with a treatise, which I would like to present to the Royal Society in Göttingen; now that I am finished, however, I do not dare to venture a sound judgment, either about its correctness in your eyes, or about whether it is worthy of being presented to the Society, and therefore I would by far prefer to leave both to your benevolent decision. Hence I submit them to you with the request, that you will be good enough to look at them at your convenience, when your time permits. . . .

With heartfelt affection and respect.

Leipzig, 1845, January 18

Your devoted,
Wilhelm Weber



Courtesy of the George August Physics Institute at Göttingen University

An exhibit honoring Gauss and Weber in June 1899 at Göttingen University. Portraits of the scientists are surrounded by their experimental apparatus and illustrations of their experiments. On the tables at left are various electrical and magnetic apparatus. The large coil in the center mounted on a wooden dolly is from the Earth inductor, which can still be seen today in the Gauss House at Göttingen.

* * *

Weber to Gauss,
No. 30, 1 February 1845

Highly honored Herr Hofrath:

I have just noticed, that in the manuscript I recently sent to you, there is apparently missing a note regarding Ampère's formula, which would be necessary in order to understand it. Namely, Ampère has given a more general expression, for the interaction of two current elements, than I introduce there, which I seek to justify, by means of the consideration that the empirically derived definition of the coefficient of the second term, which I have discarded, seems completely untrustworthy, because of the unreliability of the method, and hence that coefficient, so long as it lacks a more precise quantitative determination, by the same reasoning would have to be set = 0. If I am not in error, you yourself earlier expressed certain thoughts about discarding the negative value which Ampère assumed for that coefficient by means of which two current elements, one following the other, would have to mutually repel one another.

With heartfelt respect.

Leipzig, 1845, February 1

Your most devoted,
Wilhelm Weber

* * *

Gauss to Weber,
19 March 1845

Esteemed friend:

Since the beginning of this year, my time has been incessantly taken up and frittered away in so many ways, and on the other hand, the state of my health is so little favorable to sustained work, that up to now, I have not been in any position to go through the little treatise you were so good as to send me, and to which I just now have been able to give a first quick glance. This, however, has shown me that the subject belongs to the same investigations with which I very extensively occupied myself some 10 years ago (I mean especially in 1834-1836), and that in order to be able to express a thorough and exhaustive judgment upon your treatise, it does not suffice to read through *it,* but I would have to first plunge into study of my own work from that period, which would require all the more time, since, in the course of a preliminary survey of papers, I have found only some fragmentary snatches, although probably many more will be extant, even if not in completely ordered form.

However, if, having been removed from that subject for several years, I may permit myself to express a judgment based on recollection, I would think, to begin with, that, were Ampère still living, he would decidedly protest, when you express Ampère's law by means of the formula

$$-\frac{\alpha\alpha'}{rr}\, ii' \sin\theta \sin\theta' \cos\epsilon \qquad \textbf{(I)}$$

since that is contained in a wholly different formula, namely

$$-\frac{\alpha\alpha'}{rr}\, ii' \left(\tfrac{1}{2}\cos\theta \cos\theta' + \sin\theta \sin\theta' \cos\epsilon\right). \qquad \textbf{(II)}^2$$

Nor do I believe that Ampère would be satisfied by the appended note, which you mention in a later letter, namely, where you cast the difference in such a way, that Ampère's formula would be a *more general one,* just like

$$-\frac{\alpha\alpha'}{rr}\left(F\cos\theta \cos\theta' + G\sin\theta \sin\theta' \cos\epsilon\right)$$

where Ampère experimentally derived $F = {}^1/_2\ G$, while, because Ampère's experiments may not be very exact, you think that with equal correctness, you can claim that $F = 0$. In any other case than the present one, I would concede that in this discordance between you and Ampère, a third party would perhaps clarify the matter as follows, that:

whether one (with you) views this as merely a modification of Ampère's law, or

whether (as, in my estimation, Ampère would have to view the matter), this is nothing less than a complete overturning of Ampère's formula, and the introduction of an essentially different one,

is at bottom little more than idle word-play. As I said, in any other case I would gladly grant this, since no one can be *in* *verbis facilior* [more easy-going in matters of verbal formulation] than I. However, in the present case the difference is a vital question, for Ampère's entire theory of the interchangeability of magnetism with galvanic currents depends absolutely on the correctness of Formula II and is wholly lost, if another is chosen in its place.

I cannot contradict you, when you pronounce Ampère's experiments to be not very conclusive, while, since I do not have Ampère's classic treatise at hand, nor do I recall the manner of his experiments at all, nonetheless I do not believe that Ampère, even if he himself were to admit the incompleteness of his experiments, would authorize the adoption of an entirely different formula (I), whereby his entire theory would fall to pieces, so long as this other formula were not reinforced by *completely decisive* experiments. You must have misunderstood the reservations which, according to your second letter, I myself have expressed. Early on I was convinced, and continued to be so, that the above-mentioned interchangeability *necessarily* requires the Ampère formula, and allows no other which is not identical with that one for a closed current, *if the effect is to occur in the direction of the straight lines connecting the two current elements;* that, however, if one relinquishes the just-expressed condition, one can choose countless other forms, which for a closed current, must always give the same end result as Ampère's formula. Furthermore, one can also add that, since for this purpose it is always a matter of effects at measurable distances, nothing would prevent us from presupposing that other components might possibly enter into the formula, which are only effective at immeasurably small distances (as molecular attraction takes the place of gravitation), and that thereby, the difficulty of the repulsion of two successive elements of the same current could be removed.

In order to avert misunderstanding, I will further remark, that the Formula II above can also be written

$$-\frac{\alpha\alpha'}{rr}\, ii'\left(-\tfrac{1}{2}\cos\theta\,\cos\theta' + \sin\theta\,\sin\theta'\,\cos\epsilon\right)$$

and that I do not know, whether Ampère (whose memoire, as I said, I do not have at hand) used the first or the second notation. Both of them signify the same thing, and one uses the first form, when one measures the angle θ, θ' with the same delimited straight line; thus, this line determines the side of the second angle in the opposite way, but determines the other form, when one is considering a straight line of indeterminate length, and, for the measurement of angle θ, θ', one resorts to that line twice, in one sense or another. And, likewise, one can place a + sign in front of the whole formula instead of the − sign, if one is considering as a positive effect, not repulsion, but attraction.

Perhaps I am in a position to again delve somewhat further into this subject, which has now grown so remote from me, by the time that you delight me with a visit, as you have given me hope that you will do at the end of April or the beginning of May. Without a doubt, I would have made my investigations public long ago, had it not been the case that at the point where I broke off, what I considered to be the actual keystone was lacking

Nil actum reputans si quid superesset agendum

[Discussions accomplish nothing, if work remains to be done]

namely, the *derivation* of the additional forces (which enter into the reciprocal action of electrical particles at rest, if they are in relative motion) from the action which is *not instantaneous,* but on the contrary (in a way comparable to light) propagates itself in time. At the time, I did not succeed; however, I recall enough of the investigation at the time, not to remain wholly without hope, that success could perhaps be attained later, although—if I remember correctly—with the subjective conviction, that it would first be necessary to make a constructible representation of the *way* in which the propagation occurs.

With hearty greetings to your brothers and sister and to Professor Möbius.

Göttingen, 19 March 1845

Ever yours,
C.F. Gauss

* * *

Weber to Gauss,
No. 31, 31 March 1845

Highly honored Herr Hofrath:

Professor Buff from Giessen, who is travelling from here to Göttingen, in order to visit Woehler, his former colleague in Cassel, will have the goodness to bring you these pages. It has been of great interest to me to learn from what you were kind enough to write, that Ampère, in the definition of the coefficient he calls k in his fundamental law, was guided by other reasons, than the ones from immediate empirical experience which he cites at the beginning of his treatise, and that hence the derivation, which I first gave, because it seemed somewhat simpler, is inadmissible, because it does not reproduce Ampère's law with exactness; yet, by means of what seems to me to be a slight modification in my premise, I have easily obtained the exact expression of Ampère's law.

Through the interest taken in the matter, and through the encouragement of Fechner and later Möbius, I have been induced to occupy myself up to a point, with a subject which I conceived from the start might well be beyond me; I am all the happier that you are inclined to turn your attention once more to this arduous subject, and to give a complete development of it. Certainly, the explanation derived from a gradual propagation of the effect would be the most beautiful solution of the riddle. In response to your kind invitation, I will certainly not fail to come to Göttingen by the end of this spring.

In conformity with your instructions, I will send to the Royal Society in London a copy of the five last annual summaries of the *Resultate,* by way of the book dealer, since it will be difficult for me to pursue the invitation to Cambridge. Whence the Royal Society has obtained a copy of the first annual summary, I do not know, since they did not buy it.

Möbius, who is now celebrating his silver wedding anniversary, and my sister, remember themselves to you and your daughter with the greatest regard.

With the most heartfelt respect.

Leipzig, 1845, March 31

Your most devoted,
Wilhelm Weber

Notes

1. The title by which Weber addressed Gauss is approximately translated as "Mr. Court Councillor."
2. This seems to be Gauss's only error of memory: The epsilon should be an omega.