

Geometry

Abraham Kästner

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Definitions

1. The **limit** [**Gränze**] of a thing is its outermost, or where it terminates.
2. A **continuous magnitude** [**stetige Grösse**] is [a magnitudes whose] pieces hang together such that right where one ends the other begins, and nothing that does not belong to these quantities is in between the end of one and the beginning of another.
3. **Geometric extension** [**geometrische Ausdehnung**] is a space, which a continuous magnitude fills [up].

Scholium

It is not necessary to deal with metaphysical investigations of space and continuity here. The concept of geometric extension is an abstract concept, which remains so, however one [wie man auch sonst] might imagine these things otherwise. One must assume a continuous filling up of space, when one does not consider the particular characteristics of the thing of which the space is filled. However, on the contrary, no one is justified in subjecting the metaphysical concepts to the geometrical ones, or to attribute divisibility without end to physical extension just because it applies to geometrical extension. It applies to the latter because the general concept of division, sets no limit. Where each piece is distinguished from another through nothing other than the magnitude and position, one can make out of every part, things that do not assume exactly the same position, but have the same magnitude [Grösse], making new parts. In nature however, we find that parts distinguish themselves through more than magnitude and position alone, and thus nature somewhat hinders divisibility without end which is allowed in geometry. Natural logic, and even examples of geometry indicate that, with a general concept, determinations take place which a particular would not allow.

4. **Corporeal** [**körperliche**] **extension** of a geometric body is such an extension, which completely surrounds that which is within its limits on all sides. The extension of the body at its limit is called a surface [superfices], and the extension of the surface at its limit, a line [linea].

Scholium

A fish, which swims in the interior of a fluid, is surrounded by water and can go nowhere without driving water out of the place where he wants to go. This illustrates the idea of corporeal extension. If he swam on the surface of the water, he could carry himself over it without the water being in his way.

The words **Length**, **Breadth** and **Width** are not, according to usage, names of various, but rather of a single extension, length, which will be considered along various directions of the body. Incidentally, the extension of the surface contains nothing from the physical, which terminates with it. Therefore, surfaces cannot be piled one upon another to make up a body, just as many nothings never make up something. Similarly, no surface can consist of lines laid against one another.

5. The **point** is the limit of the line, and thus of all extension. Consequently, it itself has neither extension nor part, and an aggregate of points joined together does not constitute a line.

However, since one can cut the line anywhere at will, it consequently has points everywhere in its whole extension, and as we are generally concerned here only with the possible, it is the same whether one says: the line can have points everywhere or the line has points everywhere. Thus, one may assume points everywhere in the line. However, it is the same whether one assumes points everywhere in the line, or whether one imagines a single point, which would gradually arrive at various places in the line, since one point in no way differs from the others. Hence, it is said that a line arises from the movement of a point. However, this is not so; it consists of points lying against one another, since one cannot imagine movement without both the place which the point now assumes and the place which it assumes in the next moment, which are set at some distance from each other. But this distance is already a line. Thus the original line consists of such distances, of small lines, not of points.