[Translation in progress]

From the Measurement of Power

198. A heavy body, should it ascend from K to L (Fig. 5), must begin to climb with the known measure of the velocity at K, which it would have received in virtue of the fall through LK, because in ascent, gravity takes the same velocity [eben nehmen] from it, that it had given the body in the fall. And likewise, a body, should it ascend from Q to A (Fig.7), must begin to climb at Q with the velocity [to climb?], which the fall through AQ had given it. Thus the square of the velocities with which both bodies begin to climb, is as the height to which they ascend.

199. Thus assume that [since] the powers with which both bodies climb is as the height to which it climbs, that the body that climbs twice as high has twice as much power, then the power is as the square of the velocities; which should [they] climb once more as high, their ascent must be with the quadruple velocity, and therewith lose all power to climb, four times as much [power or height?] as that through climbing the single height.

200. Thus are both masses M ; m which initial velocity C ; c; and imagine a mass N = M, which would begin to climb with velocity c, then [verbinde man nur 199] according to 199; the evident [offenbahren] law, that with the same velocity is as the masses were; therefore

Power of M : Power of N = C^2 : c^2
Power of N : Power of m = M : m thus
Power of M : Power of m = M*C^2 : m*c^2 = $M * C^2 : m * c^2$

Or the power in the assumed significance is as the Product of the square of the velocities with their masses respectively.

.....

202. This measuring of Power had relinquished a controversy whose history alone would, as is modern mechanics, make out a volume, there seperates the greatest mathematical reasoning, and a single power has been measured by the the Cartesian, another by the leinizian type. There the leibnizian Measure of 199 assumed account of the word is apparent/evident, as one sees without going far/ (it doesn't take a genius) that the conflict must have taken the word power in different accounts. It illuminates this also from the various proofs (several proofs illuminates this), that each has led for their arguments. It has been all the same action of power calculated, hence to evaluate its powers; one had wanted to make out such action of the body true trial, one has the time, in whose action occurs, one has the time, in whose occurred the action. attracted n consideration. because the same power n another time what another action and so on. Kurzman had not argued whether power is measured though C.m or C^2.m, rather whether one should call C.m or C^2.m power. This conflict is just as to give t the right name from the Greek (Grundsprache) language, been a Logomachine; it is itself a such debate, been instructive in mathematics; than debate managed in another part of Scholarship to be. Because the versatile investigation, Releases on Duties and the like that he has courage, has really advanced our knowledge. Hence, I call some of the hither undertaken pertinent writings, and among with such, that one will have something the easiest in Germany, and be able to then instruct.

203. Johann Bernoulli's often cited *disc. Sur le mouvement* *(157. XIII) pertains exactly to this, where the Leibnizian measure of power has been vindicated through various causes. In the first volume of the Journal of the Russsian Academy of Sciences various essays concerning this controversy are found, that at that time was fashion; Such as: Bilfinger[1] *de Virib. Corp. Moto infitis & illarum mensura* p.43. Wolf's *Principia dynamica* p.217, both in favor of Leibniz. Wolf's *principiis dynamicis* countered Jac. Jurin's[2] *principia dynamica*, Phil Transact. N.479; art. 4; also Trans. N.476; art 14; communicated an investigation of the measure of power of moving bodies, and proposed a crucial essay in this controversy. Wolf has also treated the measure of power in his *Cosmologia gerneali Sect. II. C. 4.* Johann Bernoulli *de vera notione virium viuar, earunque vsu in dynamicis Act. Er. Pips. 1735; Mai p.120- Op. T. III. n. 145*. A fairly exhaustive compilation of which, what can be said concerning this, is *Dialago di Vincenzio Riccati della Compagnia di Gesu; dove ne' congressi di piu' giornate, delle forze vive e dell' azioni delle forze morte si tien disforso.* Bologna 1749; 419 Quarter pages. The work is written as a dialogue, a [Vortrag-dissertation, recitation] the Italian still loves very much, and that applies here at least well. *Della Forza de' corpi che chiamano viva libri tre, del Sigr. Francesco Maria Zanotti &c. Bol. 1752*; 311. quarter pages, contains many criticisms, especially in III B., against what Riccaati had said for the measure of vis viva. Herr Zanotti has tried, as he himself demonstrates, to define the question merely through metaphysical investigations (col sos discorso metafisico), without what is further assumed from geometry and mechanics as the most known and most common doctrines. *Felicis Anton. Balssi opusc. de viriv. viu. [das zu Bognon] 1754*. 56 quater pages. contain also remarks about Riccati. Sharp objections against the proof of the leibnizian vis viva are found in Hrn. v. Mairon's *dissertation fur l' estimation & la mesure des forces motrices des corps Nouv.Ed.Par. 1741*; wherefore *Lettre di Mr. le Mairan a Madame *** fur la question des forces vives* pertains, in which he responds to what the Marquise v. Chatelet in his *Institutions physicques* had opposed of the Leibnizian measure of power. This, with Fr. Mauquise's response is translated to German by Frau Prof. Gottschedinn , Leips. published 1742. In Leipzig are various writings (Streitschriften) on the measure of power, exchanged between the still living Hrn. Pr. Heinsius and the late M. Friedr. Wilh. Stuebner. Therein pertains the first defense disputation under the late Hausen, a 1733 *de viribus mortricibus*. Stuebner's disputation *contra virium mensuram Cartesianam, pro Leibnitiana*; 1733; as well as his *demonstratio verae mensurae virium mortricium viuar.* Heinus' *Animadersiunes*; Stuebner's *amica sepsonsio ad animaduersiones heinsii 1734*; Heins. *Notiones & discrimen virium viuar & mortuar. 1735*. The current Hr. Pr. Arnold has given a short concept of the history of this dispute, in 1754 in 2. disputations *de viribus viribus viuis earumque mensura*, thereof he defended, first under Hrn. Pr. v. Windheim, the other itself (als Praesis). Hausen has treated reaction/counteraction (Gegenwirkung) in a Leipsig 1741 published *Programma ad solemnia pronotionis magistror*, which another *ad memoriuam Geyerianam...celbrandam* followed in the same year, where he has stated/quoted the known(Gewisse) in this dispute very well. I will report the the substance of his thoughts, the best i can associate(verbinden) it with present/actual [Lehrbegriff], since according to my understanding, nothing better can be written on this subject in such brevity.

.....

On the conservation of the vis viva

205. In clause(164) this was expressed thus: with elastic bodies, the sum of the living power remains the same before and after impact. This has been called the **Conservation of Vis Viva**, and has been assumed to be a general law of nature. In 114 set; $Mx^2 + mx^2 = MC^2 + mc^2$, giving $C = c$, thus a condition, under which no impact occurs, and used the same assumption as in 110, the previous (111)

---

*Discourse on Movement

even gives c = -C. Thus the conservation of the living power does not happen with complete hard bodies. However even [if] that were one more factor/cause, such bodies are banned out of the real world.

....

On the Theory of Least Action

209. Maupertuis made known a doctrine with this name, which can be used in investigations in mechanics. Since this doctrine has caused much civil commotion, it is necessary here to give at least some account of the matter.

210. Definition. The Quantity of Action, as called by Mr. M, is the product of mass of a body with its velocity and with the space that it traverses. If a body in one position were brought to another, then the greater the action is, the greater the mass is, the faster the velocity and the longer the space thru which it goes.

211. Principle of Least Action. If a change in nature comes to pass, the least possible action is required for this change.

212. Problem. The laws of motion of hard bodies entirely derive out of the theory of least action.

solution: two hard bodies A, B follow each other with the velocities a, b each with the magnitude, which B thus from it arrived and will push. After the collision, as is easy to see, both have the same velocity, which may call x. because in a given time, with uniform motion, the space remains as the velocity, as we can also say, that both masses, have before the collision in a known time, traversed the space a, b, and traversed after the collision in the same time, each the space x. The change thus, which has thus come to pass is just so than if lasting time, there A moves itself with its velocity a thru the space, this mass, on a mathematical plane as that had moved itself this place with velocity a-x the space a-x backwards; and then if B lasting time which it moves itself thru the space b with the velocity b, from even a such surface with the velocity x-b thru the space x-b were to be led backwards. From this mathematical surface namely, gets the first of the velocity, A before the push as much as A lost in the collision. And the otherset the initial velocity from B as much, thus gained in collision. There now the motion which is with the body [loaded?] surface the same. The body may move on it rest or move itself, as are the magnitude of the actopm $A(a-x)^2$; B. $(x-b)^2$, whose sum should be a least., thus is its differential = o set; $-2aA + 2Ax + 2Bx - 2Bb = 0$ where out x = Aa + Bb / A+B
follows, as in 114.

Proof: Two hard bodies A, B follow each other with the velocities a, b, the former with the greater, which B will be reached by it and will be pushed by it. thus and will push. After the collision, as is easy to see, both have the same velocity, which may call x. because in a given time, with uniform motion, the space remains as the velocity, as we can also say, that both masses, have before the collision in a known time, traversed the space a, b, and traversed after the collision in the same time, each the space x. The change thus, which has thus come to pass is just so than if lasting time, there A moves itself with its velocity a thru the space, this mass, on a mathematical plane as that had moved itself this place with velocity a-x the space a-x backwards; and then if B lasting time which it moves itself thru the space b with the velocity b, from even a such surface with the velocity x-b thru the space x-b were to be led backwards. From this mathematical surface namely, gets the first of the velocity, A before the push as much as A lost in the collision. And the otherset the initial velocity from B as much,

thus gained in collision. There now the motion which is with the body [loaded?] surface the same. The body may move on it rest or move itself, as are the magnitude of the   actopm A(a-x)^2; B. (x-b)^2, whose sum should be a least., thus is its differential = o set; -2aA + 2Ax + 2 Bx-2Bb = 0 where out x = Aa + Bb / A+B
follows, as in 114.

213. Problem: The laws of motion of perfectly hard bodies are derived from the principle of least action.

Solution: Before a collision, the mass A traverses a space a with the velocity a, after the collision, the space alpha, with the velocity alpha; B, b, beta signify the same for the other mass as will hold for A. If further the previous consideration (212) holds, then eventually the original Action (?) A.(a-alpha)^2; B.(beta-b)^2 whose sum sets a least[simplified or minimized] , will be found to give -2Aa dalpha + 2A dalpha + 2beta d bea+ 2 B bdbata = 0.

because the collision of the elastic body can now be imagined, inasmuch as it were merely/pureley hard, and a feather beween them, which in the collision would be pressed together and again streched out, however, the feather will be pressed together with the velocity, with which the bodies approach each other, thus with their relative velocities, and consequently[just the force further expand], as with elastic bodies, the relative velocity before and after the collision is the same, thus $\beta - \alpha = a - b$ and $\beta = a - b + \alpha$, and $d\beta = d\alpha$; this value of $d\beta$ in which only established [gefundene] equations, gives first $-2Aa + -2A\alpha + 2B\beta - 2Bb = 0$ and now replace β with its value, giving

$-2Aa + 2Aa + 2Ba - 2Bb + 2B\alpha = 0$ and from that $\alpha = \dfrac{(A-B)a + 2Bb}{A+B}$ and $a - b + \alpha$ or

$$\beta = \frac{(B-A)b + 2Aa}{A+B}$$

as in (135).

214. The previous (210…..213); with only some abridgement and explainations, is translated from Mr. Maupertius's *Essay de Cosmologie*, which is in particular published in 1750 in the edition of his collected works, and [you will] find the same in book one of the Lyon 1756 published in a four octavo[1] part series *Oevres de Mr. Maupertuis*. Christlob Mylius has translated it into german; *Versuch einer Cosmologie*[2] *from Mr. Maupertuis* Berlin 1751. 8. The same thing is already in the *Mem. de l'Ac.Roy. de Pr. 1746. p.268. u.f.*[3] Since, according to this law, as much as possible will be accomplished with as little as possible, it is also called the Law of Parsimony.[4]

215. I only intended here to give a concept of Herrn von Maupertuis's law. He himself shows yet another application of it, and believes to have therein a fertile axiom of mechanics, which would itself be prefered to conservation of living power (vis viva).   Samuel König critcizes him, which had been published in Acta. Erud. March 1751. This has provoked a contention, whose circumstantial history, I, out of respect for science, would not like to publish, However, I am fairly completely provided with the thereto relevent informantion and also may posses much [that is] not very well known.

1 Paper size.
2 Essay on Cosmology
3 Memoirs of the Paris Royal Acadamy of Sciences 1746 p.268 and so on
4 Sparsamkeit

Fortunately, a greater part of this controversy did not even pertain to mechanics, and that has indeed given an opportunity, that people who would not have otherwise bothered themselves around a mathematical question have concerned themselves with this conflict. Mr. König quoted a part of a Leibniz letter, which Herr. Maupertuis took to mean that Herr König wanted to accuse him of taking Leibniz's discovery for his own, and on this account, challenged Herr. König to produce the original letter, and when he couldn't, the Royal Academy of Prussia occasioned a judgement passed on Mr. König, which came out under the title *Jugement de l'Ac. Roy. Des Sc. & fur une lettre predendue de Mr. De Leibniz to Berlin 1752*.[5]

Koenig published in opposition, a complaint under the title: *Appel au public du jugement de l'Ac. Roy.de Berlin &c*. With this occasion Voltaire fabricated against Maupertuis the *histoire du docteur Akakia*, the *Séance memorable* and other comic works. It is strange that Voltaire according to the character(see box), which would otherwise be attributed to him, had **again** avowed the Law of Parsimony. Under the title: *Vollständig Sammlung aller Streitschriften, die neulich über das vorbliche Gesetz der Natur von der kleinsten Kraft in den Wirkungen der Körper enstanden sind*[6] Leipzig 1753, a collection of these writings, emerged translated into German, which actually had nothing to do with law of least action, but rather, as I have already mentioned, took this opportunity to occupy themselves with another conflict, largely with personal attacks/personality ridden [Persönlichkeit] and insults. Informative in the main points are Mr. Eulers *diff.de principio minimae actionis Berl. 1753 and in French in the Mem. De l;Ac. De Pr. 1751 p.199*; also located on p.169 of *Harmonie entre les principes generaux de repos & de mouvem. de Mr. de Maup.* and p.246 of *essay d'une demonstration metaphysique du pr.* gen. de l' equil. In the year 1752, p.29 answer of Mr. Maupertuis against Mr. d'Arch. *Ant. Brugmans proeve over de waare grondwetten der beweging en fust [Leid] 1753* which is against Herrn. Maupertuis.

Among others, Herrn. Euler's, Additamenta to the *Methodo inueniendi curuas maximi minimiue proprietate baudentes (gens 1744)* pertains to a similar application of the maximum-minimum principle of mechanics; where he first showed how to determine the curvature of elastic plates (curvas elastica) from the consideration of maximum and minimum, then [second] what the minima or maxima will be for a function of the path of a thrown body [i.e. projectile], with no resistance.

---

5 Judgement of the Royal Acadamy of Sciences and a letter purportedly from Mr. Leibniz

6 Complete collection of all letters pertaining to the dispute, which recently arose, over the supposed law of nature of the minimum power in the action of bodies.