

A Scientific Problem: Reclaiming the Soul of Gauss

By Michael Kirsch

Before launching into his highest achievement in Book V of the *Harmony of the World*, in which he demonstrated that it is through harmonics that the physics of the solar system are known, thus redefining the nature of humanity as a whole, Johannes Kepler demonstrated that the causes of those harmonic proportions with which we measure the universe, have their origin from within the rational soul, as “*abstract quantities*”. At the height of his argument he declares:

“Finally there is a *chief and supreme argument*, that *quantities* possess a certain wonderful and obviously divine organization, and there is a shared metaphoric representation of divine and human things in them. Of the semblance of the Holy Trinity in the spherical I have written in many places.....We come, therefore, to the straight line, which by its extension from a point at the center to a single point at the surface sketches out the first rudiments of creation, and imitates the eternal begetting of the Son(represented and depicted by the departure from the center towards the infinite points of the whole surface, by *infinite lines, subject, to the most perfect equality* in all respects); and this straight line is of course an element of a corporeal form.

“If this is spread out sideways, it now suggests a corporeal form, creating a plane; but a spherical shape cut by a plane gives the shape of a *circle at its section, a true image of created mind*, which is in charge of ruling the body. It is in the same proportion to the spherical as the human mind is to the divine, that is to say as a line to a surface, though each is circular, *but to the plane, in which it is also placed, it is as the curved to the straight, which are incompatible and incommensurable*. Also the circle exists splendidly both in the plane which cuts, circumscribing the spherical shape, and in the spherical shape which is cut, by the mutual concurrence of the two, just as the mind exist in the body, giving form to it and to its connections with the corporeal form, like a kind of irradiation shed from the divine face onto the body and drawing thence its more noble nature.

“Just as this is a confirmation from the harmonic proportions of the circle as the subject and the source of their terms, equally it is the *strongest possible argument for abstraction*, as the suggestion of the divinity of the mind exists.... in a circle abstracted from corporeal and sensible things to the same extent as concepts of the curved, the

symbol of the mind, are separated and, so to speak, abstracted from the straight, the shadow of bodies.”¹

Nicolas of Cusa’s influence on Johannes Kepler in every field of his works had its origin in Cusa’s establishing the nature of the human soul’s relationship with the universe and the Creator of that universe.

This relationship addresses the greatest challenge facing mankind, and especially the youth generation today.

The nature of the universe as demonstrated in the two webpages of the LYM on Kepler, has pointed to the reality, that the principles which man discovers, never begin with necessity, or mere practical use. Science is in fact, not means to an end, but an end itself: to address the higher purpose of mankind. What is this higher purpose? In all the aims of science, mankind has been driven by an inner desire to accomplish the greatest function of the human animal; *to have fun*. Man is a creature which cannot be bounded by any bounds, because of that which lies inside man, his soul. It is in the nature of the human soul to have fun, but a certain kind, which can only be called, *real fun*.

Today the ‘Boomer’ generation filling the institutions of government and science have lost an understanding of how to have *real fun*; in doing so, they have misplaced a thorough conception of their own souls. Since they lack this freedom, they also fail to understand the deeper implications of science, and its relation to humanity. The effect of an entire generation having lost the conception of the immortality of the human soul, has been a dynamic and multilayered collapse of the U.S. and world economy, the U.S. institutions of Government, and a rabid empiricism which dominates science. Therefore, given the need and possibility of such events as the recent Russian proposal for joint U.S.-Russia cooperation on the Bering straits project, what is required today is a clear conception.

Three months ago, and none too soon, a sea change occurred in modern science; the elaboration by the LYM of Kepler’s achievement in actually redefining the potential of the human species, the human soul, and the nature of all human knowledge, put modern empiricism on notice and has shaken the rotting foundations of current thinking. This revolution in science sparked by the Kepler Two project, must continue so that a new generation of economic scientists are unleashed who do not fail to bring the essence of the human soul as defined by Kepler in *The Harmony of the World* fully into the domain of modern science.

In a fantastic irony, the needed challenge for such a change in science intersects the specific task of this of this report: *The third phase of ‘Animating Creativity’ on Gauss begs the question: by what means, might we discover the thought process that allowed Carl Gauss to discover the orbit of Ceres?* Understanding the principles he did discover, and comparing them with the method employed in his 1799 *Fundamental Theorem of Algebra*, it is furthermore clear that Gauss greatly obscured the nature of his thoughts

¹ Johannes Kepler, *The Harmony of the World*, Book IV Chapter 1

throughout almost all his work. The Napoleonic tyranny that swept Europe, and later the cultural collapse of Romanticism following the Congress of Vienna, were the conditions in which Gauss decided to take such a course.² However, since the nature of ‘harmonics’ as discovered uniquely by Kepler, must be carried forward and applied to the domain of modern science, the implications of Carl Gauss’ discoveries and the thinking he had concerning them, *must* be fully comprehended.

To this end, there are no means more suitable for such an immortal task—in reviving the nature of mankind in science today, and the consequences which that implies—than to study the mind of Nicolas of Cusa and his student, Kepler, whose relationship released the Earth in motion from the shackles of empiricism, and with it all of modern science. In carrying forward the scientific revolution of Cusa and Kepler, without losing the freedom of thinking involved in the completely integrated epistemology contained therein, the hidden genius of Gauss will become accessible. In other words, how did Cusa and Kepler think, as reflected in what is explicit in their work—which can be a guide to reflect back onto Gauss’s work—thereby drawing out the substance of what was implicit in his unspoken thoughts?

Abraham Kästner, the architect of the German renaissance and the teacher of Carl Gauss, considered Nicolas of Cusa to be a founder of many fields of science, which preceded the work of many, including Kepler and Leibniz. This is cause for celebration, and also indicates the great likelihood of Gauss’ acquaintance with Cusa’s ideas.

Therefore, what we now show is how the discoveries of Cusa and his conception of the human soul, took root in Johannes Kepler, and today provides the basis for discussing Carl Gauss’ elaboration of: an anti-Euclidean harmonic solar system, his comprehension of the transcendental nature of the Kepler Problem, the applications of the method of Leibniz’ infinitesimal in his discovery of the orbit of Ceres, and above all, his contribution to the ‘higher purpose’ of mankind.

Part I: The Edifice of the World

Abraham Kästner, in 1757, in his *Praise of Astronomy* declared Nicolas of Cusa to be, one of two “revivers of the edifice of the world” along with Copernicus.³ Although Cusa was a close collaborator of Toscanelli, who was a famous astronomer at the time, the most probable reference is

² Tarranja Dorsey, *First Thoughts on the Determination of the Orbit of Gauss*
<http://www.wlym.com/~animations/ceres/PDF/Tarranja/OrbitOfGauss.pdf>

³ <http://www.wlym.com/~animations/ceres/PDF/Tarranja/KaestLobderSternk.pdf>

to Cusa's *De Docta Ignorantia*. In that work, there lies a principle so vast, that its implications will guide us through the entirety of this investigation.

Nicolas of Cusa sought to demonstrate that the Creator of the Universe was not something able to be reduced to a particular metaphor or described in any way, but only known inconceivably by the mind of man, and that all knowledge sought and captured by man came from seeking after this knowledge of the Creator. Cusa investigated the nature of such a universe, that which he calls a "contracted maximum", as the medium between the absolute infinite and the plurality of finite things. Here he returns the conception of the universe to the Pythagorean conception of forms, which make up the 'world soul' in a universe which is not a duality, as defined by Aristotle, of, on the one side, unknowable principles and, on the other, the world of the changeable sense, but instead a universe with an infinite Creator whose perfection reaches through the universe to all matter. Although there are many paradoxes he sets forward concerning how the idea of a maximum existing in plurality is known, we go here to the heart of the issue.

In the course of investigating the Absolute Maximum—a subject to which we will return—he makes the following observation: of things admitting of more or less, we never come to an unqualifiedly maximum or minimum. Therefore, he states, since only the cause of all causes, is the Maximum, and is the only absolute infinite not subject to being greater or lesser by any degree, we never come therefore to Absolute Equality, except in the Maximum. That is, only the Maximum which contains all things in it, including the minimum, is equal to itself. Since only in the Maximum is found absolute Equality, all things differ. From this comes an immortal statement by Cusa:

"Therefore, one motion cannot be equal to another; nor can one motion be the measure of another, since, necessarily, the measure and the thing measured differ" and "With regard to motion, we do not come to an unqualifiedly minimum".

What implications did this hold for astronomy?

"...It is not the case that in any genus— *even [the genus] of motion*—we come to an unqualifiedly maximum and minimum. Hence, if we consider the various movements of the spheres, [we will see that] it is *not possible for the world-machine to have, as a fixed and immovable center, either our perceptible Earth or air or fire or any other thing*. For, with regard to motion, we do not come to an unqualifiedly minimum—i.e., to a *fixed center*. For the [unqualifiedly] minimum must coincide with the [unqualifiedly] maximum; therefore, the center of the world coincides with the circumference. Hence, the world does not have a [fixed] circumference. For if it had a [fixed] center, it would also have a [fixed] circumference; and hence it would have its own beginning and end within itself, and it would be bounded in relation to something else, and beyond the world there would be both something else and space (*locus*). But all these [consequences] are false. Therefore, since it is not possible for the world to be enclosed between a physical center and [a physical] circumference, the world—of which God is the center and the circumference— is not understood. And although

the world is not infinite, it cannot be conceived as finite, because it lacks boundaries within which it is enclosed.⁴

“Therefore, *the Earth, which cannot be the center, cannot be devoid of all motion. . . .*

Therefore, just as the Earth is not the center of the world, so the sphere of fixed stars is not its circumference. . . .

“*And since we can discern motion only in relation to something fixed, viz., either poles or centers, and since we presuppose these [poles or centers] when we measure motions, we find that as we go about conjecturing, we err with regard to all [measurements]. And we are surprised when we do not find that the stars are in the right position according to the rules of measurement of the ancients, for we suppose that the ancients rightly conceived of centers and poles and measures.*

“*. . . Neither the sun nor the moon nor the Earth nor any sphere can by its motion describe a true circle, since none of these are moved about a fixed [point]. Moreover, it is not the case that there can be posited a circle so true that a still truer one cannot be posited. And it is never the case that at two different times [a star or a sphere] is moved in precisely equal ways or that [on these two occasions its motion] describes equal approximate-circles—even if the matter does not seem this way to us.*”⁵

Here in these passages Cusa, deriving the universe as a product of a Maximum Creator with a certain paradoxical relation to the universe, derived principles, which are seen today, after the work of Johannes Kepler, to be entirely true. The universe which is infinite with respect to all things is such that it even coincides with the minimum. And if we are talking about the boundary of the universe, it is such that the center coincides with the circumference. Since motion never comes to a minimum, there is no fixed center; not even the sun is completely devoid of motion. Thus the Aristotelian Ptolemaic model system was exposed as a fraud.⁶ This truth would be thoroughly demonstrated by Kepler in refuting the Equant.⁷ Cusa moved the Earth out of a fixed center, and set it into motion, an idea which would later be taken up by Copernicus. Cusa sets up the paradox that since all motion is derived from the comparison with something fixed, all astronomical

⁴ Since it is not the maximum, the universe could have been greater, but since in the possibility of being, matter cannot be extended unto infinity, the universe could not be greater. Thus it is unbounded and with respect to all that can be in actuality, nothing is greater than it.

⁵ In *De Ludo Globi* Cusa, discussing the motion of the irregularly shaped ball used for the game, and the conditions of the ground, and the way in which each different player sets the ball on the ground, says “It is not possible to do something the same way twice, for it implies a contradiction that there be two things that are equal in all respects without any difference at all. How can many things be many without a difference? And even if the more experienced player always tries to conduct himself in the same way, this is nevertheless not precisely possible, although the difference is not always perceived.” Abraham Kästner in his review of Cusa says that this is Leibniz’ Principle of Indiscernibility. <http://wlym.com/ceres/~animations/PDF/Michael/history1.pdf>

⁶ For Kepler’s discussion of the Aristotelian and Ptolemaic model, See Part I of Kepler’s *New Astronomy*

⁷ March 2007 Vol. 1 No. 3 www.seattlelym.com/dynamis

knowledge of his time is thrown into error, since the platform of observations is itself moving. This would later be taken up by Kepler in calculating the orbit of the Earth in Chapters 22-30 of the *New Astronomy*.⁸ Cusa also established that since motion never occurs around a fixed point, there are no perfect Circles.⁹ This was left for Kepler to demonstrate in Chapters 40-60 of the *New Astronomy*.¹⁰ Likewise the non-circular orbits are constantly adjusting themselves to a different center, and thus cause the orbits of the bodies to take a different course. Lastly, Cusa did away with the idea that there is a limit to the universe, at the “eighth sphere” of the fixed stars.

Thus a constantly changing universe was established, with no fixed center. Within such an ‘imprecise’ universe with no place devoid of motion, how could the cause of motion be determined, as motion was derived from more than simply comparing two objects, with one at rest? This higher concept of motion was left untouched until Kepler established the true physical causes in the *New Astronomy* in chapters 32-40.¹¹

Part II: What is Science?

What therefore is man that he exists within such a universe? How must mankind approach the challenge of a universe, which, as Cusa says, is a “contracted” image of the Absolute Maximum, in which imprecision enters into all considerations of measurement? Therefore, how does the human mind then, proceed to investigate the causes in such a universe?

In Nicolas of Cusa’s *De Docta Ignorantia*, he begins by stating that all things desire to exist in the best manner possible, and use their judgment so that this desire is not in vain, allowing each

⁸ <http://www.wlym.com/~animations/newastronomy.html> Part III

⁹ In Cusa’s *Theological Compliment* he proves again why there can be no perfect circles, referencing back to his *De Docta Ignorantia*. Kepler is reported to have most certainly read this work. See Commentary Notes on Chapter II in *The Mysterium Cosmographicum*, and Eric Aiton, ‘Infinitesimals and the Area Law’ in F.Kraft, K.Meyer, and B.Sticker, eds., Internationales Kepler Symposium Weill der Stadt, 1971(Hildesheim, 1973) p. 286 Given Kepler’s knowledge of this fact he most likely already knew what to look for when arriving at Tycho Brahe’s house in 1600.

¹⁰ <http://www.wlym.com/~animations/newastronomy.html> Part IV

¹¹ <http://www.wlym.com/~animations/newastronomy.html> Part III This higher understanding of motion was also the central question in Leibniz determination of a dynamic, rather than the fraud of Descartes. The following quote from Leibniz’ 1692 *Critical Thoughts on the General Part of the Principles of Descartes*: “If motion is nothing but the change of contact or of immediate vicinity, it follows that we can never define which thing is moved. For just as the same phenomena may be interpreted by *different hypotheses in astronomy*, so it will always be possible to attribute the real motion to either one or the other of the two bodies which change their mutual vicinity or position. Hence, since one of them is arbitrarily chosen to be at rest or moving at a given rate in a given line, we may define geometrically what motion or rest is to be inscribed to the other, so as to produce the given phenomena. Hence if there is nothing more in motion than this reciprocal change, it follows that there is no reason in nature to ascribe motion to one thing rather than to others. The consequence of this will be that there is no real motion. Thus, in order to say that something is moving, we will require not only that it change its position with respect to other things but also that there be within itself a *cause of change, a force, an action.*”[emphasis added]

being to attain rest in what they seek. With the power of number, mankind judges the uncertain, proportionally, by comparing it with the certain. Cusa states an apparent paradox that arises:

“Both the precise combinations in corporeal things and the congruent relating of known to unknown surpass human reason to such an extent that Socrates seemed himself to know nothing except that he did not know. . .”

If we were created with a desire to seek knowledge and given only these means of comparative relation, then, a paradox seems to arise. If all we come to know in our seeking is that we don't know, weren't we created in vain?

Rather, we must desire to know that we do not know!

“No! It's a trap,” an Aristotelian shouts, “don't you see? This proves that you can't know anything about the invisible universe. All you can do is assume *a priori* and set up set of definitions and axioms that follow. Forget about whether the initial axiom is knowable, it will work!” Somewhere, a baby boomer sighs relief, “Thank goodness, you alerted me, I thought I was going to have to think to get past this one. I like beliefs so much better. They just feel right, you know?”

Instead, Cusa concludes “If we can fully attain unto this knowledge or our ignorance, we will attain unto learned ignorance. ... The more he knows that he is unknowing...the more learned he will be.”

Now, after wrestling with this, ask the question: if we seek to become learned in our ignorance, what must humans study, to attain the maximum learning of our ignorance?

Cusa proceeds, bringing us with him to measure the *Maximum*, to that very end. But how can you measure the absolute Maximum? If measuring is done by means of comparative relations, what can be compared to the absolute Maximum? There is no comparative relation of the finite to the infinite. Things greater or lesser partake in finite things, and the maximum does not. The “rule of learned ignorance”¹² is that in things greater something can always be greater, in things lesser, always lesser, thus in comparing two things we never find them to be so equal that they could not be more equal indefinitely.

Cusa elaborates the paradox which the intellect faces with such an incomprehensible maximum. Since the maximum is not greater or lesser, it is both maximally large, and maximally small, or the minimum, thus the maximum is such that it coincides with the minimum. Since the maximum is not greater or lesser, it does not allow opposition, there are no opposites in the maximum, and therefore, he states what appears to be logically inconsistent: “Thus the Maximum is beyond all affirmation and negation: it is not, as well as is, all things conceived to be, and is as well as

¹²<http://cla.umn.edu/sites/jhopkins/DeLudo12-2000.pdf> Book II section 96

is not, all things conceived not to be. It is one thing such that it is all things, and all things such that it is no thing, maximum such that it is minimum.”¹³

But how can such contradictions be combined? If we are created to seek maximum ignorance, but such a maximum only creates inconsistencies in our understanding, how can the human intellect not have been created in vain? Cusa—throwing Aristotle’s maxim “each thing either is or is not” out the window—stated that infinite truth must therefore be comprehended not directly, by means comparisons of things greater or lesser, but, rather, “incomprehensibly comprehended!”¹⁴

To proceed further toward our end, Cusa then declares, spinning Aristotle in his grave¹⁵:

“We must leave behind the things which, together with their material associations, are attained through the senses, through the imagination, or through reason-[leave them behind] so that we may arrive at the most simple and most

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1. ¹³ *De Docta Ignorantia* Book I, Chapter 4 Cusa continues to elaborate the characteristics of the Maximum in the following chapters. He continues on to say that everything is limited and bounded with a beginning and an end, and so all finite things never proceed to infinity because then infinity would be reduced to the nature of finite things, and thus the Maximum is the beginning and end of all finite things. Every finite thing is originated, because it could not come from itself, because it would then exist when it didn’t.

In *De Ludo Globi*[F3], he similarly demonstrates the necessity for the maximum, stating that since all things must be something, and all things *exist*, and in all existent things there is *being*, without which they couldn’t exist, so, therefore, the *being* of all things is present *in* all *existing* things, and all *existing* things exist *in being*. Thus the most simple being is the exemplar of all existing things, and this exemplar, the being of all things, or Absolute Being, is the Creator of all existing things, for the exemplar of something generates that something as an image of itself. Therefore, nothing exists without Absolute Being.

¹⁴John Wenck accused Cusa of asserting that absolutely nothing could be known. Cusa replied in his *Apologia Doctae Ignorantiae* “For in an image the truth cannot at all be seen as it is [in itself]. For every image, in that it is an image, falls short of the truth of its exemplar. Hence, it seemed to our critic that what is incomprehensible is not grasped incomprehensibly by means of any transcending. But if anyone realizes that an image is an image of the exemplar, then leaping beyond the image he turns himself incomprehensibly to the incomprehensible truth. For he who conceives of each creature as an image of the one Creator sees hereby that just as the being of an image does not at all have any perfection from itself, so it’s every perfection is from that of which it is an image; for the exemplar is the measure and the form (*ratio*) of the image.”

Cusa had been sent to Constantinople as part of his attempts to reunite the Greek and Roman Churches. He returned in February 1438. At the end of the *De Docta Ignorantia*, Cusa states, “while I was at sea en route back from Greece,¹⁴⁶ I was led (by, as I believe, a heavenly gift from the Father of lights, from whom comes every excellent gift)¹⁴⁷ to embrace—in learned ignorance and through a transcending of the incorruptible truths which are humanly knowable—incomprehensible things incomprehensibly.”

¹⁵ Aristotle in his metaphysics, after a lengthy attack on the Pythagorean conception of number states in his final conclusion: “the objects of mathematics are not separable from sensible things, as some say, and they are not the first principles.”

abstract understanding, where all things are one, where a line is a triangle, a circle, and a sphere, where oneness is threeness (and conversely), where accident is substance, where body is mind (*spiritus*), where motion is rest, and other such things.”

In conducting an inquiry into unseen truths, visible images must be used to reflect the unseen as a mirror or metaphor. However, for the visible image to truly reflect the invisible, there must be no doubt about the image.¹⁶

As Cusa said before, the mind invokes comparative relations of the known to the unknown to come to knowledge. But all perceptible things are in a state of continual instability because of the material possibility abounding in them. For example, when a geometer uses mathematical figures for measuring things he seeks not the lines in material, as he cannot draw the same figure twice, but seeks the line in the mind. For perceptible figures are always capable of greater precision, being variable and imperfect. Cusa says that the eye sees color as the mind sees its concepts, but the mind sees more clearly, as insensible things are unchangeable.

As Plato said:

“And do you not also know that [geometers] further make use of the visible forms and talk about them, though they are not thinking of them but of those things of which they are a likeness, pursuing their inquiry for the sake of the square as such and the diagonal as such, and not for the sake of the image of it which they draw?... The very things which they mold and draw, which have shadows and images of themselves in water, these things they treat in their turn as only images, but what they really seek is to get sight of those realities which can be seen only by the mind.”¹⁷

The triangle in the mind, which is free of perceptible otherness, is therefore the triangle which is the truest. Cusa says the Mind is to the mathematical figures it contains, as forms are to their images. Then, since mathematical things in the mind are the forms, and thus do not admit of otherness, the mind could be said to be the form of forms.

The mind views the figures in its own unchangeability. “But its unchangeability is its truth. Therefore, where the mind views whatever [figures] it views: there the truth of it itself and of all the things that it views is present. Therefore, the truth wherein the mind views all things is the mind’s form. Hence, in the mind a light of- truth is present; through this light the mind exists, and in it the mind views itself and all other things.”¹⁸

¹⁶ Abraham Kästner remarks on the importance of this concept in his review of Cusa’s *De Venatione Sapiente* See Translations from the Geschichte <http://wlym.com/~animations/ceres/PDF/Michael/history1.pdf>
<http://wlym.com/~animations/ceres/PDF/Michael/history2.pdf>

¹⁷ Book VI Plato’s *Republic*

¹⁸ Nicolas of Cusa *Theological Compliment*

But, since truth is the form of the mind, it is not something greater or lesser, and thus as it is a Maximum to the mind, it is not seen directly. Cusa likens the truth to an invisible mirror in the mind. And as is the rule of learned ignorance, that which is not the maximum can always be a greater or lesser; that which is not truth can never measure truth so precisely that it couldn't surpass the former measure. "Now, the mind's power is increased by the mind's viewing; it is kindled as is a spark when glowing. And because the mind's power increases when from potentiality it is more and more brought to actuality by the light-of-truth, it will never be depleted, because it will never arrive at that degree at which the light-of- truth cannot elevate it more highly."¹⁹

But wait, since our desire to know everything about the universe clashes with the Maximum truth being infinitely distant, then logically wouldn't the Creator be evil?

In truth, there is nothing more fun, as Cusa perfectly describes:

"Moreover, that movement is a supremely delightful movement, because it is a movement toward the mind's life and, hence, contains within itself rest. For, in moving, the mind is not made tired but, rather, is greatly inflamed. And the more swiftly the mind is moved, the more delightfully it is conveyed by the light-of-life unto the Mind's own life."²⁰

Therefore, although the view of the likes of Norbert Wiener and his information theorist followers claim that mankind is in a race against entropy, and will never be able to discover everything fast enough, making them "[S]hip wrecked passengers on a doomed planet."²¹; in truth, this paradox of the mind's inability to comprehend the entire universe, is not part of an evil design, it is in fact what drives the universe forward. The speculation of mankind is not a sign of an entropy of the mind, but is the nourishment itself, and in the process of mankind's discoveries, the universe develops.²²

Since this is the purpose of mankind's nature, to ascend with the intellect, Nicolas of Cusa demonstrated that the universe itself is a reflection of this relationship of the mind of man and the universe as a whole. The comparison for how the mind seeks the truth in measuring the 'Maximum Number' was demonstrated in Cusa's extensive treatment of the relationship of the curved and straight, which formed the basis for all of modern science, and the ascent of which we will no longer prolong.

Part III: On the Curved and Straight

¹⁹ Ibid.

²⁰ Ibid.

²¹ Norbert Wiener *Human Use of Human Beings* Chapter II Progress and Entropy

²² Norbert Wiener *The Human use of Human Beings*

“As Cusa’s criticism of the error of Archimedes on the subject of the isoperimetric principle expressed by the circle, echoes the relevant conception, the cognitive power of the specifically human individual mind is not a secretion of the living body, but a principle which subsumes the living body *dynamically*. This dynamical principle of human reason, reflects the idea of the image of the Creator.”-Lyndon LaRouche, *Cusa and Kepler*

Nicolas of Cusa demonstrated a fundamental truth about the nature of the curved and straight. The mind’s attempt to relate the curved and the straight represents its capability to measure the universe as a bounding array of Maximum numbers, which once identified—and distinguished in the same way as the human mind is distinguished from the Maximum—could be incomprehensibly comprehended.

In Cusa’s on the Quadrature of the Circle he begins:

“There are scholars, who allow for the quadrature of the circle. They must necessarily admit, that circumferences can be equal to the perimeters of polygons, since the circle is set equal to the rectangle with the radius of the circle as its smaller and the semi-circumference as its larger side. If the square equal to a circle could thus be transformed into a rectangle, then one would have the straight line equal to the circular line. Thus, one would come to the equality of the perimeters of the circle and the polygon, as is self-evident.”²³

Cusa states that the central premise of Archimedes is : *since one can have a greater or a lesser polygonal perimeter, then one can have also an equal perimeter.*

Those who followed Archimedes thought therefore, says Cusa,

“If the square that can be given is also not larger or smaller than the circle by the smallest specifiable fraction of the square or of the circle, they call it equal. That is to say, they apprehend the concept of equality such that what exceeds the other or is exceeded by it by no rational—not even the very smallest—fraction is equal to another.”

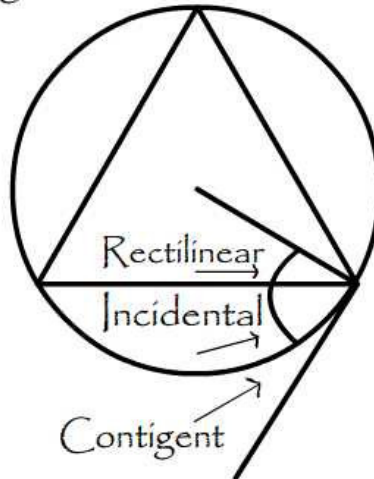
But, Cusa says, there were those who disagreed that where one can give a larger and a smaller, one can also give an equal. This applies to the angles which arise in the relations of the circle and polygon. As he continued:

“There can namely be given an incidental angle that is greater than a rectilinear, and another incidental angle smaller than the rectilinear, and nevertheless never one equal to the rectilinear. Therefore with incommensurable magnitudes this conclusion does

²³ All Quotes in this Section unless otherwise indicated are taken from Nicolas of Cusa’s *On the Quadrature of the Circle* Translated by Will Wertz. http://www.schillerinstitute.org/fid_91-96/941_quad_circle.html

not hold. That is to say, if one could give one incidental angle that is larger than this rectilinear angle by a rational fraction of the rectilinear, and another that is smaller than this rectilinear by a rational fraction of the rectilinear, then one could also give one equal to the perimeter. *But since the incidental angle is not proportional to the rectilinear, it cannot be larger or smaller by a rational fraction of the rectilinear, thus also never equal.* And since between the area of a circle and a rectilinear enclosed area there can exist no rational proportion.... Therefore the conclusion is also here not permissible.”[emphasis added]²⁴

Angles



Cusa had challenged this already in his *De Docta Ignorantia*. “[T]here can never in any respect be something equal to another, even if at one time one thing is less than another and at another [time] is greater than this other, it makes this transition with a certain singularity, so that it never attains precise equality [with the other]. ...And an angle of incidence increases from being lesser than a right [angle] to being greater [than a right angle] without the medium of equality.”²⁵

See Animation :

<http://wlym.com/~animations/ceres/PDF/Michael/Moving%20Incidental%20Angle.swf>

The nature of the incidental angle compared to the rectilinear angle drives the point home, that *if the circle could be converted into the polygon*, then each of the parts of the circle and each of the parts of the rectilinear polygon could be a part of the other, but a segment of the circle cannot be transformed into a rectilinear area because of the nature of the incidental angles.

²⁴ This question of incidental angles was a great epistemological debate with grand implications. See Will Wertz’ *Nicolas of Cusa’s ‘On the Quadrature of the Circle’* http://www.schillerinstitute.org/fid_97-01/012_Cusa_quad_circ.html

²⁵ *De Docta Ignorantia* Book III Chapter I

After showing this incommensurability of the curved and straight angles, Cusa concludes the point:

“If a circle can be transformed into a square, then it necessarily follows, that its segments can be transformed into rectilinearly enclosed figures. And since the latter is impossible, the former, from which it was deduced, must also be impossible.”

Thus, the following property of the circle arises:

“Just as the incidental angle cannot be transformed into a rectilinear, so the circle cannot be converted into a rectilinearly enclosed figure.”

But how close could you get? Cusa says there is an incommensurability between the two kinds of angles, but what exactly is it?

Just how close can one get to precision, and why is absolute precision impossible with the curved and straight? To demonstrate this Cusa says that if one uses the contingent angle—a *very* small angle—it is possible to give: 1) an incidental angle smaller than a rectilinear angle by the contingent angle, which is not any rational fraction of the incidental angle and 2) a rectilinear angle larger than the incidental angle by a contingent angle which is also not any rational fraction of the rectilinear.

That is an incidental angle + contingent angle = rectilinear angle
a rectilinear – contingent angle = incidental angle

But wait a second—Cusa says the contingent angle “is not a rational fraction of the incidental or contingent angle.” One cannot add and subtract incommensurable magnitudes to attain equality.

See Animation: <http://wlym.com/~animations/ceres/PDF/Michael/Moving%20Contigent.swf>

In the same way he says, one can give a square that is larger in a perimeter by the circle, yet not by a rational proportion of the square, and one can give a smaller circle than a square, yet not by a rational proportion of the circle. Therefore a smaller and larger square can be given to the circle but never come so close which is smaller or larger by a rational fraction.

As he said in *De Docta Ignorantia*, “Similarly, a square inscribed in a circle passes—with respect to the size of the circumscribing circle—from being a square which is smaller than the circle to being a square larger than the circle, without ever arriving at being equal to the circle.”²⁶

He then remarks on what necessarily follows.

In ‘On conjectures’ Cusa had identified what the nature of a numbers such as the circle were: “Hence, species are as numbers that come together from two opposite directions—[numbers] that proceed from a minimum which is maximum and from a maximum to which a minimum is not opposed.”²⁷

²⁶ *De Docta Ignorantia* Book III Chapter I

²⁷ Nicolas of Cusa *On Conjectures*

He also states here in the *On the Quadrature of the Circle*:

“In respect to things which admit of a larger and smaller, one does not come to an absolute maximum...” and since “polygonal figures are not magnitudes of the same species...” a polygon never becomes small enough or large enough to equal a circle. “Namely, in comparison to the polygons, which admit of a larger and smaller, and thereby do not attain to the circle’s area, the area of a circle is the absolute maximum, just as numerals do not attain the power of comprehension of unity and multiplicities do not attain the power of the simple.”

The more angles the inscribed polygon has, the more similar it is to the circle. However, even if the number of its angles is increased ad infinitum, the polygon never becomes equal to the circle unless it is resolved into an identity with the circle.”

The Characteristic of Learned Ignorance

All of the above in this section was the gist of Cusa’s overview as to what the nature of the problem is. Afterwards, Cusa identifies the degree of incommensurability that exists when seeking for the isoperimetric circle. It is as though: although he identified the incommensurability between the different angles, he had yet to identify the degree of imprecision that exists. What follows therefore, is Cusa’s elaborate process of setting up incommensurable proportionals to box in the nature of the species difference.

Isoperimetric means: equal perimeter. In the *Mathematical Compliment*, the idea of isoperimetric takes a broader meaning, in looking at triangles and squares and other polygons that all have equal perimeter, and what the relationship of the radius’ would be that circumscribe those figures.

Here, in *On the Quadrature of the Circle*, Cusa is looking for the radius of the circle whose perimeter would be equal to the perimeter of a give triangle which is inscribed in a circle. Where would such a radius be? What would be its characteristics?

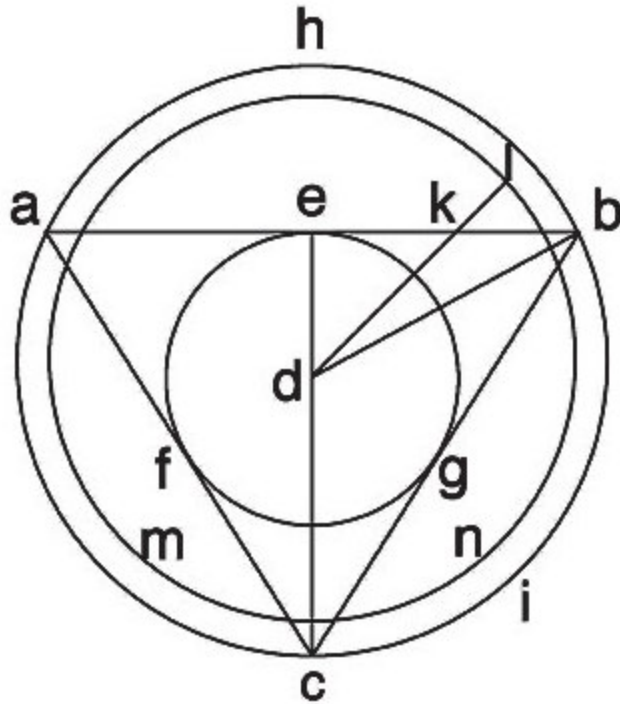
See Animation: <http://wlym.com/~animations/ceres/PDF/Michael/QofC2nd.swf>

First, he shows that the simple idea of an equality between the triangle perimeter and the circular perimeter creates a paradox which yields the defining characteristic of the isoperimetric radius. This provides the pathway to box in where it must dwell.

To demonstrate the equality of the circular to the triangular perimeter, he had to show that the “radius must be to the sum of the sides of the triangle, as the radius of the [isoperimetric] circle is to the circumference.” But—and here is the crux—since the radius has no rational proportion to the circumference, such a radius would not be proportional to the sides of the triangle, because if the radius is to the circumference, and if the triangular circumference were equal to the circle, then it would share in the lack of proportionality with the radius.

See Animation: <http://wlym.com/~animations/ceres/PDF/Michael/QofCIncPer.swf>

The sought line, the radius of the isoperimetric polygon, cuts the side of the triangle. But what follows from the above statement is, that since it is not proportional to the circumference of the polygon, so it would not be proportional to any part of it, or proportional in square to any part of it. Therefore, in this diagram, since the radius of the isoperimetric circle we are looking for, dl , is not proportional to the perimeter of the triangle, then also the line dk —which would be proportional to dl — would not be proportional to eb , de , or db . Nor would the line ek , created by dk , be proportional to eb , de , or db



And what this points to, is an extremely important affirmation by Cusa. Since, as was shown, no line can be drawn that stands in rational proportion with the sides of the triangle, *no point on eb could be given precisely that the ‘sought length’ would be drawn to.* Thus, any length along eb , which is in proportion to eb , would not be in proportion to the length sought. And also, any length which is drawn from d such that it would be in proportion to a length along eb , would not be the ‘sought length’.

So this gives us the method of approach to boxing in our isoperimetric radius right? Since the sought line is not proportional to eb and db , what we are looking for then, must be to find the line which is the *most non-proportional* to them, and then, we will have the line which is the *least non-proportional* to the ‘sought length’. The length we are looking for compared to the lengths that are known, those of the triangle, is the minimum with respect to its degree of knowability. Therefore, we are looking for the radius which brings us the most ignorance relative to the known triangle!

Where must the cut be? One extends the length that cuts the line, by the proportion of the line on the side of the triangle—created by the cutting line—to the whole side of the triangle[see animation] and also the line on the other side of the cut to the whole side. However, since the line cutting the line has to be proportional to the one we are looking for, the extension must also be proportional. But, the line drawn to the side of the triangle from d can never be exactly proportional

to the one sought since the sought length is not proportional to the sides of the triangle. It cuts it larger or smaller. So if it extends it by the proportion of the side of the triangle, its extension can never be exact either. So which extension is least non-proportional to the one sought?

The fact that we can find a length that is smaller than the one sought, and one larger than the one sought, means there should be a length where we can cut the line such that it is neither larger nor smaller, right? The closest we can come, Cusa says, is when both extensions are equal to each other and thus the amount by which the created length is larger or smaller than the sought length is the smallest it can be, even though it is not the sought length by the amount smaller or larger but not by a rational fraction; again, because of the incommensurability between the isoperimetric radius and the perimeter of the triangle.²⁸

See Animations: <http://wlym.com/~animations/ceres/PDF/Michael/inscribed%20triangle.swf>
<http://wlym.com/~animations/ceres/PDF/Michael/Pi.swf>

After finding the closest value for the isoperimetric radius, he makes his point:

“True, that is not the precise value, but it is neither larger nor smaller by a minute, or a specifiable fraction of a minute. And so one cannot know by how much it diverges from ultimate precision, since it is not reachable with a usual number. And therefore this error can also not be removed, since it is only comprehensible through a higher insight and by no means through a visible attempt. From that alone you can now know, that only in the domain inaccessible to our knowledge, will a more precise value be reached. *I have not found that this realization has been passed along until now.*”[emphasis added]

At the conclusion, having thus demonstrated what he called a ‘species’ difference, which even Archimedes failed to see, Cusa remarks on the ‘higher purpose’ of seeking truth.

“The measure with which man strives for the inquiry of truth has no rational proportion to Truth itself, and consequently, the person who is contented on this side of precision does not perceive the error. And therein do men differentiate themselves: These boast to have advanced to the complete precision, whose unattainability the wise recognize, so that those are the wiser, who know of their ignorance.”

²⁸ As an example non-proportionality between magnitudes, he says that the lines bounding the incidental, rectilinear, and coincidental angles share in the non-proportionality that their angles share, magnitudes which are larger or smaller than each other by a magnitude larger or smaller than a rational fraction. This line he says is “before all divisibility of the line...by which a straight line can cut a straight line in two.... It is like an unattainable endpoint [of a line]...nonetheless...in its way, divisible by a curve.” The point he makes is that the normal divisibility of a line which lies between two endpoints is different than the divisibility of the line bounding the contingent angle, and yet it is still divisible *in its way*. This contingent angle length is the difference between proportionality and non-proportionality. This magnitude is the type which describes how close one can approach the sought length.

Mathematics of the Infinite

Later, in the *Theological Complement*, Cusa introduces the needed conceptions that the ancients were missing. It was not that they presupposed the coincidence in equality of the circle and square, which Cusa says all seekers do,²⁹ but that they endeavored to manifest what they presupposed by means of reason. “But they failed *because reason does not admit that there are coincidences of opposites.*”³⁰

“But the coincidence of those features which are found to be diverse in every polygon ... ought to have been sought *intellectually*, in terms of a circle; and [then those inquirers] would have arrived at their goal.”

Having demonstrated the species difference of the circle, Cusa introduced the exact method of approach to the ‘incomprehensible maximum’ in the *De Docta Ignorantia*, again, here, in the case of this maximum ‘number’ indicated by the species difference.

From the *De Docta Ignorantia*:

“But since from the preceding [points] it is evident that the unqualifiedly Maximum cannot be any of the things which we either know or conceive: when we set out to investigate the Maximum metaphorically, we must leap beyond simple likeness.”³¹ In other words, to represent the infinite, which bounds all things, we must move from mathematical relations in the finite, to mathematical relations in the infinite, and only then compare these infinite mathematical figures to the absolute infinite.

For it is the nature of the intellect to conceive of such infinite relations, as the mind itself conceives everything in such a way. When a mathematician draws a triangle or circle, he looks to the infinite exemplar. The triangle drawn is actually infinite in the mind, and not subject to size. The triangle that is imagined in the mind, it is not thought of as large or small, it is not imagined as 4 feet, 10 feet, or 1000 feet, but as the potential of all triangles.

Applying the rule of learned ignorance from the *De Docta Ignorantia*: any curve which admits of more or less cannot be a maximum or minimum curve. And measuring a curve with the rule of learned ignorance, we see that the maximum curved line is straight, and the minimally curved line is straight, therefore, a curve is in reality nothing but partaking in a certain amount of straightness to a

²⁹ Cusa said that the knowledge is presupposed, to which the mind is guided by a light of truth in the mind. And all who seek knowledge are instigated by that infinite art or science.

³⁰ All Quotes in this section are taken from Nicolas of Cusa’s *Theological Compliment*

³¹ *De Docta Ignorantia* Book I Chapter 12

greater or lesser degree. Now comparing the curved and straight, the straight line participates more in the infinite line than a curved line participates in it.³²



To See Animation: <http://wlym.com/~animations/ceres/PDF/Michael/infinitecircle.swf>

Then Cusa says: “At this point our ignorance will be taught incomprehensibly how we are to think more correctly and truly about the Most High as we grope by means of a metaphor.” In the *Theological Compliment*, with this “Most High” number, Cusa applied this method of the infinite to a true solution of the quadrature of the circle. Cusa shows that the relations between the circle and polygons is only comprehended in the infinite, that in the infinite all polygons coincide with the infinite circle

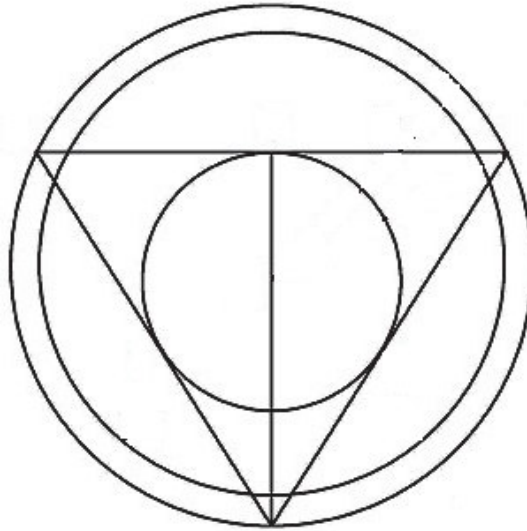
His point is best expressed in the two different responses to the following question: *how do you find the perimeter of a circle, whose measure is a straight line?*

Archimedes reply was to use an exhaustive method of approximation and he failed to grasp the higher concept.

Cusa however, answered the question as follows: “We come to the truth of the equality of curved and straight only through considering the isoperimetric circle as triune through the coincidence of opposites in polygons...The triune isoperimetric circle is the coincidence of three circles in which the perimeter of the circle is found whose measure is a straight line. In such a circle,

³² Cusa says on this topic “the most congruent measure of Substance and accident is the Maximum.” Leibniz later demonstrated this issue of substance, that if the predicates were *in* the substance, then a clear concept was had of the substance. (As Cusa says, the Creator creates, and Man forms conceptions of the created. The clearest concept of the substance is when nothing interferes with predicate’s expression of the substance, as is the case of the catenary curve, as the most clearest expression of the principle of least action, as shown in Leibniz construction of the catenary which most clearly expresses the irony of the paradox of physical action; that is, the complex domain. Therefore the implications of Cusa’s principle of Maximum- Minimum were developed further in the infinitesimal calculus.

the inscribed circle and circumscribed coincide... and the polygon in the middle too.”



What is Cusa talking about? His point is, that real isoperimetric circle is in the infinite. The solution exists in the intellect, where the relations between different species becomes clear. The infinite brings the boundaries of a species into the understanding, thereby illuminating the concept of a generating principle.

Cusa had made this point in the *De Docta Ignorantia* as he brought the infinite to mathematics. Cusa used the example of the infinite line to demonstrate that the maximum is in all things and all things are in the maximum. Each finite line could be divided endlessly and yet, a line would always remain. Thus the essence of the infinite line was in finite line. Likewise each line, when extended infinitely, became equal, whether it was 4 feet or 2 feet. Thus the essence of each finite line was in the infinite line, although participated in by each finite line in different degrees. Here, similarly in the maximum, the circle is *in* every polygon, in such a way that each polygon is *in* the circle. “*The one is in the other, and there is one infinite perimeter of all.*”

Cusa concludes the discussion of his solution as such:

“The ancients sought after the squaring of a circle....If they had sought after the circularizing of a square, they might have succeeded. . . a circle is not measured but measures.... [I]f you propose to measure the maximal truth ...as if it were a circular line—you will be able to do so only if you establish that some circular line is the measure of a given straight line.”

“Given a finite straight-line, a finite circular-line will be its measure. Thus, given an infinite circular-line, an infinite straight-line will be the measure of the infinite circular-line.... Because the infinite circular-line is straight, the infinite straight-line is the true measure that measures the infinite circular-line...Therefore,

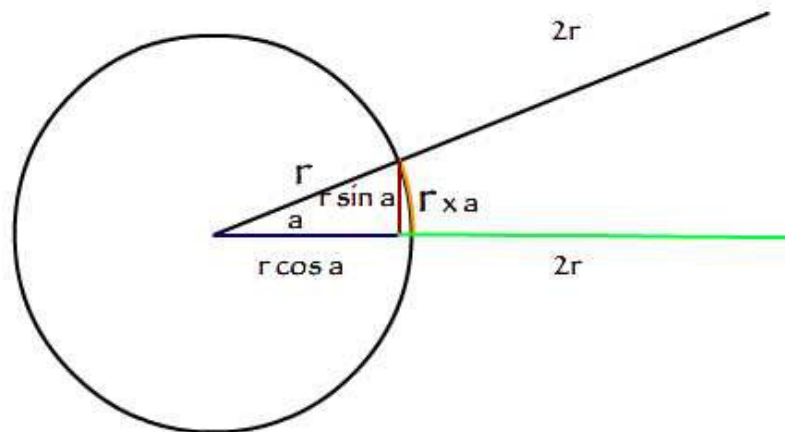
the coincidence of opposites is as the circumference of an infinite circle; and the difference between opposites is as the circumference of a finite polygon.”³³

Infinitesimals?

In Cusa’s *Mathematical Perfection* whose aim was “to hunt for mathematical perfection from the coincidence of opposites,” he investigates whether the smallest chord of which there cannot be smaller were not as small as its arc. Cusa says, as learned ignorance teaches, since neither the chord nor the arc could become so small that they could not become smaller, both are capable of being smaller, “since the continuum is infinitely divisible.”

³⁴

Cusa Says: the half Arc is to the half Chord, as triple the radius, is to the sum of the Cosine plus twice the radius
 $r \times a : r \sin a = 3r : r \cos a + 2r$



To See Animation: <http://wlym.com/~animations/ceres/PDF/Michael/kastneranimation.swf>

³³Nicolas of Cusa’s *Mathematical Complement* is not available in English, thereby making many of the mathematical theorems in the *Theological Complement* very vague. Among them is the following: “There cannot be found a straight line equal to a circular line, unless first the opposite is found i.e. a circular line equal to a straight line. Once this is found, then, from a proportion between circular lines, the unknown straight line is found, through both the known line and known proportion of circular lines...T here can be exhibited a circular line that is equal to a given straight line, but not conversely. For only if the former equality is known can the latter equality be known—and then [only] as proportionally [equal], as is explained in my oft-mentioned book *Complementum*.”

³⁴ Kästner’s Review of Cusa’s Geometrical writings Translated by Michael Kirsch
<http://wlym.com/ceres/PDF/Michael/history1.pdf>

At the end of Cusa's *Mathematical Perfection*, after investigating the minimal arc of a circle to determine the relation between the half arc and sine³⁵, he states:

“In a similar manner, you yourself may derive the relationship with regard to the minimum in other curved surfaces. What can be known in mathematics in a human manner, from my point of view, can be found in this manner.”³⁶

In what is historically of great importance, Abraham Kästner, in his review of Cusa's works, remarked about this statement:

“That sounds like bringing in the infinitesimal calculus (analysis of the infinite). Thus one could say something to the cardinal which he had not considered. In fact, he contemplated evanescent magnitudes, only he did not know how this conception would be used.”³⁷

Infinitesimal: Imprecise Measure for the Transcendental

Lyndon LaRouche in his Paper *For Today's Youth: Cusa and Kepler* wrote:

“Cusa's treatment of the circle, in correcting the error of Archimedes, is... of crucial clinical significance, in our search for insight, for our reaching out in our zeal to touch the substance of the human soul within ourselves, or in others.”

Cusa's investigation of the curved and straight is a model for the identification of the nature of the human soul. It is more than a simple likeness. There is no other way to ascend to the identification of species differences in magnitude. It is the capability of the human mind, to conceive and discover the relations between transcendental magnitudes through ascending to the intellect and in viewing as if through a mirror, the image of a higher principle reflected in the intellect as a species difference, and comprehended incomprehensibly. The transcendental magnitude delivers mankind to an understanding of power, an understanding of universal principles which express themselves to the visible domain as an image of creativity.

³⁵ Cusa had also stated in *On Conjectures* Part II Chapter II “For if every chord is smaller than the arc that it subtends, and if the chord of a smaller arc is more like its own arc than the chord of a larger arc [is like its arc], then if we were to admit that the two chords of the half-arcs were equal to the chord of the whole arc, it would be evident that a coincidence of chord and arc would be implied.”

³⁶ Ibid.

³⁷ Ibid.

Cusa concluded in his *Quadrature of the Circle* with this discussion, “And they are entities that have a circular, interminable movement around the being of the infinite circle. They encompass within themselves the power of all other species on the path of assimilation, and, beholding everything in themselves, and viewing themselves as the image of the infinite circle and through just this image—that is, themselves—they elevate themselves to the eternal Truth or to the Original itself. These are creatures bestowed with cognition, who embrace all with the power of their mind.”

Indeed, for Nicolas of Cusa, the relation of the curved and straight was no mere comparison, as such, that is, not a case of “this is like that.” Nicolas of Cusa saw every human as conceiving in their mind an infinite circle, which is the measure of all things, as an image of the absolute maximum. All finite things, all expressions of number, every polygon, and every other shape is measured by this eternal conception of the infinite circle. The intellect being continually guided forward by this exemplar in the mind toward ever higher understanding of how this measurement reveals the truth in all things.

Cusa saw the form of circular movement precedes all circular movement and is altogether free of time. The form of the circle is seen in reason, which exists in the rational soul. But where is reason except in the rational soul? Therefore, if the soul sees within itself the form of the circle, which is beyond time, then it must be beyond time. Thus it cannot cease or perish.³⁸

Part IV: Unfolded Implications

Cusa’s higher understanding of the purpose of mathematics, was fully alive in the mind of Kepler. Kepler also found that these conceptions and demonstrations of Cusa were necessary to continue forward to a higher understanding of the universe. Many of his discoveries were influenced by Cusa’s thinking. Here we take a look at the broad range of such discoveries keeping the question in mind: what implications did it have for Gauss’ discovery of the orbit of Ceres?

Kepler’s conception of the entire universe was shaped most prominently by Cusa; particularly on the question of ‘quantity’. In his *Mysterium Cosmographicum*, in the Second Chapter, before putting forward his conception of the nested Platonic solids as the organization of planets, it is Cusa’s curved and straight which guides the way:

“It was matter which God created in the beginning.....I say what God intended was *quantity*. To achieve it he needed everything which pertains to the essence of matter; and *quantity* is a form of matter, in virtue of its being matter, and the source of its definition. Now God decided that *quantity* should exist before all other things so that there should be a means of comparing a curved with a straight line. For in this one respect Nicholas of Cusa and others seem to me divine, that they attached

³⁸ For more on Cusa’s conception of the human soul, See [Appendix: Cusa On the Human Soul](#)

so much importance to the relationship between a straight and a curved line and dared to liken a curve to God, a straight line to his creatures; and those who tried to compare the Creator to his creatures, God to Man, and divine judgments to human judgments did not perform much more valuable a service than those who tried to compare a curve with a straight line, a circle with a square.

“.....To this was also added something else which is far greater: the image of God the Three in One in a spherical surface, that is of the Father in the center, the Son in the surface, and the Spirit in the regularity of the relationship between the point and the circumference.....Nor can I be persuaded that any kind of curve is more noble than a spherical surface, or more perfect. For a globe is more than a spherical surface, and mingled with straightness, by which alone its interior is filled.

“But after all why were the distinctions between curved and straight, and the nobility of a curve, among God’s intentions when he displayed the universe? Why indeed? Unless because by a most perfect Creator it was absolutely necessary that a most beautiful work should be produced.

“This pattern, this Idea, he wished to imprint on the universe, so that it should become as good and as fine as possible; and so that it might become capable of accepting this Idea, he created *quantity*; and the wisest of Creators devised quantities so that their whole essence, so to speak, depended on these two characteristics, straightness and curvedness, of which curvedness was to represent God for us in the two aspects which have just been stated.....For it must not be supposed that these characteristics which are so appropriate for the portrayal of God come into existence randomly, or that God did not have precisely that in mind but created *quantity* in matter for different reasons and with a different intention, and that the contrast between straight and curved, and the resemblance to God, came into existence subsequently of their own accord, as if by accident.

“It is more probable that at the beginning of all things it was with a definite intention that the straight and the curved were chosen by God to delineate the divinity of the Creator of the universe; and that it was in order that those should come into being that *quantities* existed, and that it was in order that *quantity* should have its place that first of all matter was created.....³⁹

In various letters of Kepler he expressed the same sentiment concerning Cusa’s view of man:

“Geometry is one and eternal, a reflection out of the mind of God. That mankind shares in it is one of the reasons to call man an image of God.”

³⁹ Johannes Kepler’s *Mysterium Cosmographicum* Chapter II

“Man’s intellect is created for understanding, not of just anything whatsoever but of *quantities*. It grasps a matter so much the more correctly the closer it approaches pure *quantities* as its source. But the further something diverges from them, that much more do darkness and error appear. It is the nature of our intellect... the study of divine matters concepts which are built upon the category of quantity; if it is deprived of these concepts, then it can define only by pure negations.”

“No eerie hunch is wrong. For man is an image of God, and it is quite possible that he thinks the same way as God in matters which concern the adornment of the world. For the world partakes of quantity and the mind of man grasps nothing better than *quantities* for the recognition of which he was obviously created.”⁴⁰

Later in Kepler’s investigation of Light in his *Optics* in 1604, again this influence of Cusa concerning the curved and straight and his conception of the infinite sphere, would present itself as the opening conception concerning the relationship of space :

“For when the most wise founder strove to make everything as good, as well adorned and as excellent as possible..... [there] arose the entire category of quantities, and within it, the distinctions between the curved and the straight, and the most excellent figure of all, the spherical surface. For in forming it, the most wise founder played out the image of his reverend trinity. Hence the point of the center is in a way the origin of the spherical solid, the surface the image of the inmost point, and the road to discovering it. The surface is understood as coming to be through an infinite outward movement of the point out of its own self, until it arrives at a certain equality of all outward movements. The point communicates itself into this extension, in such a way that the point and the surface, in a commuted proportion of density with extension, are equals.⁴¹ Hence, between the point and the surface there is everywhere an utterly absolute equality, a most compact union, a most beautiful conspiring, connection, relation, proportion, and commensurateness. And since these are clearly three—the center, the surface, and the interval—they are nonetheless one, inasmuch as none of them, even in thought, can be absent without destroying the whole.....The sun is accordingly a particular body, in it is this faculty of communicating itself to all things, which we call light....⁴²

Infinitesimal Considerations

40

⁴¹ In Cusa’s *De Docta Ignorantia* Book I, Chapter 23 he said “The center of a maximum sphere is equal to the diameter and to the circumference. ... for in an infinite sphere the center, the diameter, and the circumference are the same thing.”

⁴² Kepler’s *Optics* Chapter I

However, although Cusa discovered the method to investigate the Maximum, i.e. universal principles, he did not indicate how these principles express themselves at every moment of change.

But, as Kästner remarked that Cusa's investigation in his *Mathematical Perfection*⁴³ appeared to be introducing infinitesimals into the construction, one wonders, what influence did this have on Kepler's discovery of such magnitudes?

Kepler, moving beyond geometry, into the domain of physics discovered the form in which the motion along the orbit expresses the unseen physical principle at every moment. Kepler had found out he was wrong in the small, by 8' of an arc. But in order to correct it, he had to know the whole orbit.

Working on calculating the motion of the Earth, Kepler, in Chapter 32 of the *New Astronomy*, as he is deriving the principle that the time needed to traverse an arc of the orbit is inversely proportional to the distance from the sun, states: "But since[the daily arc of the eccentric at aphelion] and [the daily arc of the eccentric at perihelion]are taken as minimal arcs they do not differ appreciably from straight lines." Why did he do this? Kepler was the first to discover the principles of planetary motion. They were not self evident! In order to know the whole orbit, he had to discover the relationship expressed at each moment. Thus, in thinking how to represent a path that reflects the power of the Sun, he conceived of the idea of using 'minimal arcs' that represent moments of a process of continual change along the orbit.⁴⁴ Kepler was able to determine the *whole* orbit by understanding the relationship expressed in the smallest possible *part* of the orbit.

Leibniz later generalized the method for the actual physical actions of the universe so that the infinite may be accessible to the human mind. Leibniz showed with the calculus, that the many physical curves which he and the Bernoullis investigated were the reflection of a unseen physical principle, a dynamic, which represented itself as knowable to the human mind in the form of an infinitesimal relationship, as a metaphor for that dynamic. At his highest point, after exposing the fraud of Cartesian physics by posing the challenge of the curve of isochronous descent, he then discovered the complex domain, a higher geometry in which the action of physical principles could be represented.⁴⁵

As we work forward in Gauss' discovery, the reader should keep in mind, that his use of such magnitudes of higher order, has its basis in Cusa's ideas, and Kepler's first application, and later

⁴³ Kepler is also said to have certainly read this work. Eric Aiton, 'Infinitesimals and the Area Law' in F.Kraft, K.Meyer, and B.Sticker, eds., Internationales Kepler Symposium Weill der Stadt, 1971(Hildesheim, 1973) p. 286

⁴⁴ Gauss in his *Summary Overview* very often finds himself dealing with higher order magnitudes. Similar to Kepler, he swapped curved areas with straight areas in the small. In the *Summary Overview*, g represents the sector of an orbit between to positions of a heavenly body and the sun, and f represented the triangle formed between those two observations and the sun. In one calculation, Gauss stated, "We can set $f : g' = 1$, since the difference is only of the second order."

⁴⁵ For more on Leibniz' Calculus see October 2006 Vol. 1 No. 1 www.seattleym.com/dynamis

curvedness, the latter pure straightness. The hyperbola, parabola and ellipse are placed in between, and participate in the straight and the curved, the parabola equally, the hyperbola in more of the straightness, and the ellipse in more of the curvedness. For that reason, as the hyperbola is extended farther, it becomes more similar to a straight line, i.e. to its asymptote.⁴⁶ The farther the ellipse is continued beyond the center, the more it emulates circularity, and finally it again comes together with itself. ...the lines drawn from these points touching the section, to their points of tangency, form angles equal to those that are made when the opposite points are joined with these same points of tangency. For the sake of light, and with an eye turned towards mechanics, we shall call these points "foci".⁴⁷

While investigating the hyperbola and the relation between the chord and the sagitta, as the focus moves closer to the base, he says "The sagitta⁴⁸ ... is ever less and less until it vanishes and the chord at the same time is made infinite since it coincides with its own arc(speaking improperly since the arc is a straight line.)"⁴⁹

Echoing the infinite metaphors of Cusa, he continues: "For geometrical terms ought to be at our service for analogy. I love analogies most of all: they are my most faithful teachers, aware of all the hidden secrets of nature. In geometry in particular they are to be taken up, since they restrict the infinity of cases between their respective extremes and the mean with however many absurd phrases, and place the whole essence of any subject vividly before the eyes."⁵⁰

The Transcendental

Lastly, and perhaps of greatest importance is the foundation of the transcendental magnitude discovered by Cusa and its contribution to the 'higher purpose' of mankind.

⁴⁶ What implications did this have for Gauss' later use of this continuity of conic sections in the *Theoria Motus*? In an interesting echo of this sentiment Gauss also treats the parabola as an infinite ellipse. "If the parabola is regarded as an ellipse, of which the major axis is infinitely great . . ."

⁴⁷ Kepler's *Optics*, Chapter on Conics

⁴⁸ In the diagram the sagitta it is the length A, the focus of the hyperbola, to S on the axis of the hyperbola.

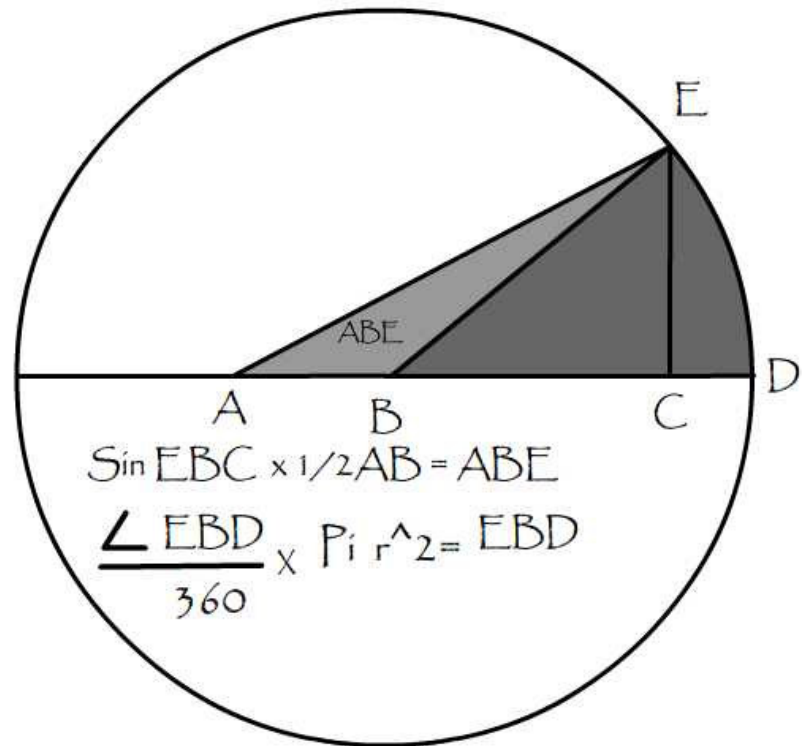
⁴⁹ Ibid.

⁵⁰ Ibid.

"[Given] the mean anomaly, there is no geometrical method of proceeding to the equated, that is, to the eccentric anomaly. For the mean anomaly is composed of two areas, a sector and a triangle. And while the former is numbered by the arc of the eccentric, the latter is numbered by the sine of that area multiplied by the value of the maximum triangle, omitting the last digits. And the ratios between the arcs and their sines are infinite in number. So, when we begin with the sum of the two, we cannot say how great the arc is, and how great its sine, corresponding to this sum, unless we were previously to investigate the area resulting from a given arc; that is, unless you were to have constructed tables and to have worked from them subsequently."

- Johannes Kepler *New Astronomy* Chapter 60

$$ABE + EBD = \text{Mean Anomaly}$$



51 52

For the question arises, what was Kepler's Problem? What did he do which caused such ferment after his death? Why was there a political operation to get rid of his Problem?⁵³ Reflect on

⁵¹ Cusa identified the nature of the species difference in the Quadrature of the Circle. His solution to 'rectify' the curved line, was to apply his method of coincidence of opposites with the maximum circle. How would Cusa's method be applied to solve the Kepler problem, which expresses the inability to relate the arcs and sines? Further how does Cusa's method therefore lead into the higher functions of Gauss and Riemann which address the Kepler Problem?

⁵² Transcendental equations and magnitudes are employed and encountered by Gauss throughout the *Theoria Motus*. Gauss discusses the Kepler Problem, and makes advancements toward solving the problem. In one location there, Gauss remarks that it is possible to determine the whole orbit by two radii vectors if their magnitude and position are given together with the *time* taken to move from one radii vector to the next (between the two positions). But, "This problem", he says, "considered among the most important in the theory of the motions of the heavenly bodies, is not so easily solved, since the expression of the time in terms of the elements is transcendental..."

⁵³ Peter Martinson *Neither Venetians or Empiricists Can Handle Discoveries*
<http://www.wlym.com/~animations/ceres/PDF/Peter/Astronomy.pdf>

Cusa's discussion of the nature of the human mind's relationship to infinite truth as the true relation of curved and straight.

Above all, *this* was Kepler's 'problem'. It was the 'problem' which led him to seek the relationship between the physical causes and the true motions of the planets.

After Kepler succeeded in demonstrating the physical cause of the motions of the planets, he then ventured forth to correlate that cause with the motions. This required not merely associating a known principle with observations; the power of the species from the sun caused motions of the planets to express themselves in the form of the countless paradoxes of Chapters 41-60 and led Kepler into an unexplored domain of the mind. And only by the passion with which he chased after it, with a presupposition of the truth, willing to become sufficiently knowledgeable of his ignorance, did Kepler succeed in relating the unseen principle to the sense perceptions—of the observations, the distances, and equations—and brought the understanding of his intellect into actuality. And while the unseen principle was finally brought into visible distance with the mind's eye, and seen to take the form of an ellipse, even this was still a shadow of a paradoxical motion of a higher power, a 'maximum' truth, which was unknowably knowable in the form of the same species identified by Cusa: the transcendental nature of the arc and sine.

The Newtonians, in their attempt to reduce transcendental magnitudes to lower algebraic magnitudes with their infinite series, in their attempt to bury Kepler's 'Problem' had already been proved wrong by Cusa.⁵⁴

"Number is always greater or lesser and never one, for then it would be the maximum or minimum number and then, number, being all things, would necessarily no longer be multiple but absolute oneness, therefore, the Maximum must be that minimum and maximum number, One."⁵⁵

In other words, one never can come to the Maximum number through an infinite succession of numbers, because then number would cease to exist, and "all finite things never proceed to infinity because then infinity would be reduced to the nature of finite things."⁵⁶

However, the true intention in banning the 'Kepler problem' was to outlaw such thinking as Kepler's, for this higher paradox served as a mirror of our own likeness to the image of the Creator, driving mankind toward the infinite truth.

⁵⁴ John Keil claimed to have 'solved' the Kepler Problem with an infinite series.

⁵⁵ *De Docta Ignorantia* Book I Chapter VI

⁵⁶ *Ibid.*

Part V: An Imprecise Harmony

In the *Harmony of the World*, Kepler took these questions of the human soul and geometry as we discussed earlier, and the issue of imprecision was approached again, this time with the harmonies, and he accomplished the greatest furthering the relation of man and the universe, in a more profound way than ever, moving beyond simply the curved and straight as expressed in his solids, to the moving form of the soul itself in the heavens.

In Book I of *Harmony of the World*, Kepler discovered the causes of the harmonic proportions mathematically, as no one had ever done before, and developed how these quantities are intellectual, knowable, and derived from the mind. Before Kepler, they were studied as something outside the mind.⁵⁷ The only divisions of a circle which are ‘knowable’ to the human mind, turn out later in Book III to also be the only divisions of a string which are harmonic to the human ear.⁵⁸ Thus, with such a relationship to Nicolas of Cusa, through all of his work, it should be no surprise that before launching into Book V of his *Harmony of the World*, he looked to Cusa’s conception of the curved and straight to demonstrate that the proportions of the harmonies had their foundation in the nature of man as in the image of the Creator.⁵⁹ As he said: “Finally there is a *chief and supreme argument*, that *quantities* possess a certain wonderful and obviously divine organization, and there is a shared metaphoric representation of divine and human things in them...”⁶⁰

With these harmonies established as proportions from the soul, Kepler then took up his edifice of the world from his *Mysterium* and bringing together his *New Astronomy*, sought to demonstrate the causes of the motions. Kepler determined the extreme motions of the planets at Perihelion and Aphelion as the area to seek harmony in the heavens, and proceeded to calculate every possible proportion between each of the planets’ diverging, converging, and extreme motion in pairs. Once he then fit the planet’s harmonies to the musical scale, he went on to determine the origin of the eccentricities of the planets and also, to look at the Solar system as a harmonic whole.

As soon as Kepler began to organize the Solar System as a whole as one harmonic system, in the second part of Chapter Nine in *The Harmony of the World*, the echo of Cusa’s principle of ‘imprecision’ in the universe—with which we began this investigation—could be heard.

⁵⁷ Johannes Kepler *The Harmony of the World*, Introduction to Book I, Book I, and Book IV

⁵⁸ What does it mean, that the reason why proportions are harmonic, and why they sound ‘musical’ to the human ear, is because they are knowable to the human mind? What does this say about the human mind? Is it looking as from outside the universe, analyzing sense perceptions from the outside, or rather, from within?

⁵⁹ Also, See Chris’ Article on Book IV

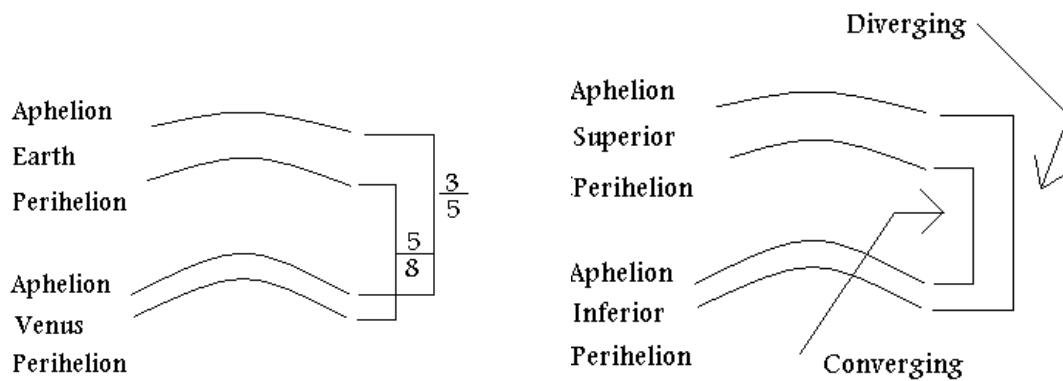
⁶⁰ The reader is encouraged to return to the beginning of this article, where the entire quote was placed.

“Conformably to the rule, there is no precision in music. Therefore, it is not the case that one thing [perfectly] harmonizes with another in weight or length or thickness. Nor is it possible to find between the different sounds of flutes, bells, human voices, and other instruments comparative relations which are precisely harmonic— so [precisely] that a more precise one could not be exhibited. Nor is there, in different instruments [of the same kind]—just as also not in different men—the same degree of true comparative relations; rather, in all things difference according to place, time, complexity, and other [considerations] is necessary. And so, precise comparative relation is seen only formally; and we cannot experience in perceptible objects a most agreeable, undefective harmony, because it is not present there. Ascend now to [the recognition] that the maximum, most precise harmony is an equality-of-comparative-relation which a living and bodily man cannot hear. For since [this harmony] is every proportion (*ratio*), it would attract to itself our soul's reason [*ratio*] — just as infinite Light [attracts] all light—so that the soul, freed from perceptible objects, would not without rapture hear with the intellect's ear this supremely concordant harmony. A certain immensely pleasant contemplation could here be engaged in—not only regarding the immortality of our intellectual, rational spirit (which harbors in its nature incorruptible reason, through which the mind attains, of itself, to the concordant and the discordant likeness in musical things). But also regarding the eternal joy into which the blessed are conducted, once they are freed from the things of this world.”⁶¹

For, in proposition XXVI of chapter nine, while constructing the intervals between Venus and Earth, Kepler ran into such ‘imprecision’. In propositions XXIII-XXV he developed the fact that the characteristics necessary to have a solar system with both hard and soft melody depended on the hard sixth, 3/5, between their aphelial motions, that is Venus’ aphelion and Earth aphelion, and a soft sixth, 5/8, between their perihelial motions. This created the necessity for very small changes to each planets own individual motions. He said that “harmonic beauty” urged that these planets’ own motions—that is, the proportion between one planet’s perihelion and aphelion—since they were very small and cannot be any of the harmonic intervals, should at least be of the melodic intervals, that is the diesis 24/25, or the semitone 15/16.⁶² But in this case, Kepler had shown that the two intervals of Earth and Venus’ own motion would have to differ by a diesis in themselves, these two melodic intervals the 24:25 and 15:16, differ by 125:128, which is smaller. Therefore, Kepler showed that only one of the planets could have the melodic interval. Either the Earth would have the semitone, 15:16, and Venus the 125:128, a non-melodic interval, or Venus would have the diesis 24:25, and Earth would have 12:13 a non-melodic double diesis.

⁶¹ *De Docta Ignorantia*, Book II Chapter I

⁶² On Harmonic versus Melodic Intervals, See LYM Harmonies Website
<http://wlym.com/~animations/harmonies/site.php?goto=melodic.html>



“But since the two planets have equal rights, therefore if the nature of melody had to be violated in their own proportions, it had to be violated equally in both cases, so that the difference between their own intervals could remain exactly a diesis, to differentiate the necessary kinds of harmonies... Now the nature of melody was equally violated in both cases if the factor by which the superior planet’s own proportion fell short of a double diesis, or exceeded a semitone, was the factor by which the inferior’s own proportion fell short of a simple diesis, or exceeded the interval 125:128.”⁶³

So instead of the Earth’s motion having either the melodic semitone of 15:16 or the unmelodic interval of 12:13, it has 14:15, and instead of Venus having the melodic diesis of 24:25 or the unmelodic interval of 125:128, it had 35:36. And 14:15 and 35:36, both differ from 15:16 and 24:25 by 80:81, a musical comma! Cusa identified the universe as one of ‘imprecision’, in which the physics of orbits of planets were in a state of continual change, but Kepler has identified the method to make this ‘imprecision’ knowable. The continuous change expressed itself in the form of a comma. The comma is not a ‘thing’ but occurs—as in other places in Chapter 9 of the *Harmony of the World*—as a consequence of the musicality of the system as a whole. Here the musicality of the system, in the region containing the key to both kinds of harmony, soft and hard, demanded the dissonance be spread out equally, which took the form of a comma.⁶⁴

⁶³ *Harmony of the World* Book V, Chapter 9 Proposition XXVI

⁶⁴ Although more is needed to demonstrate it, this also points to question: is the relationship between the orbits of planets transcendental? Rianna St. Classis discussed this question in the LYM *Harmony of the World* website: “The harmonic nature of the relationship of the individual planets and the sun is reflected in the total orbital period of each planet, the total area of the orbit swept out as equal areas in equal times, or better, as Kepler views it, the area swept out by the planet *is* the time it has traveled. This is echoed in the fact that within an individual orbit, at two moments, the proportion of the apparent (from the sun) speeds has an inverse relationship to the proportion of the squares of the distances of the planet from the sun at those moments. But this relationship does not hold *between* planets. If the area a

And in the face of those who would demand a fixed universe, those who would argue, “Well aren’t you just fudging this? Aren’t you accepting this small change just to impose *your* hypothesis onto the universe?” Kepler, understanding the nature of imprecision of a universe based on change said:

“Do you ask whether the highest creative wisdom would have been taken up with searching out these thin little arguments? I answer that it is possible for many arguments to escape me. But if the nature of harmony has not supplied weightier arguments... it is not absurd for God to have followed even these, however thin they may appear, since he has ordered nothing without reason. “For it would be far more absurd to declare that God has snatched these quantities, which are in fact below the limit of a minor tone prescribed for them, accidentally. Nor is it sufficient to say that He adopted that size because that size pleased Him. For in matters of geometry which are subject to freedom of choice it has not pleased God to do anything without some geometrical reason or other, as is apparent in the borders of leaves, in the scales of fishes, in the hides of wild beasts, and in their spots and the ordering of their spots, and the like.”⁶⁵

Kepler’s method of hypothesis cures the mental diseases of entropy found so common in modern science today. The human soul’s own proportions found throughout create the circumstances that we are inside the universe, and that we understand it as a reflection of ourselves. This thinking is exactly opposite to the empiricism that struck Europe after the death of Leibniz.

The underlying axiom of science today is immediate skepticism at one’s mind’s ability to know the reason for the creation of the universe. And so when a human discovers such intricacies as the comma, which create a harmonic organization, the immediate reaction is to say, “Well, this universe may be harmonic, but, it sure held together pretty thinly. You’re telling me it hangs on the difference of 15/16 to 12/13 to 14/15? And 9/10 to 24/25, to 35/36? You must be imposing your assumptions on to this.”

Rather than looking at such matters, and remarking at the absolute perfection that exists, and celebrating in the mind’s capability, there is the fear of the popular ideal that there is no God in science, and thus, we are imposing our thoughts onto the universe.⁶⁶ Such thinking is entropic,

planet sweeps out is the time it has traveled, this time is unique to this individual planet. 100 units of Mars’s orbit *are not* equal to 100 units of Jupiter’s orbit. If we were to evaluate these two portions from the standpoint of how we think of time on the earth, according to the earth’s rotation about its axis, the number of days Mars took to travel 100 units would be different than the number of days Jupiter took to travel 100 units.”

⁶⁵ Ibid.

⁶⁶ The Case of Leibniz’ discovery of the catenary principle is an example of the folly of modern thinking concerning science, and an example which irreparably dooms the credence of its modern ways. Leibniz and Bernoulli demonstrated that the change in direction at every possible moment of a curve, is guided by a constant physical relationship between vertical and horizontal tension, i.e., the physical differential relationship. However, Leibniz, who had launched a scientific political movement against the Cartesians, had turned physics into a problem of finding the dynamic, i.e. the individual substance, determining the effects. Therefore, he sought more than the physical relationship guiding the

because in that thinking one must force the universe into harmony, one has to put it together piece by piece, and it is delicately holding together, rather than the idea that one is on the inside of it, *and have detected in the small the reason for its perfection*. Such imprecisions as commas and infinitesimals are not seen as a fragile argument that needs to be held together with great convincing, but are the reflection of the relationships indicating a new unseen dynamic.

Inquire further. How did Kepler determine the causes for the eccentricities? Did the physics of the orbital elements randomly create harmony, or did the necessity for harmony generate each orbit as it is? Further, if each orbit necessitated creating harmony, how did the Solar System become one whole harmonic system? Take a few examples for the relations of the Solar System as a whole.

Kepler investigates why Earth and Venus have the smallest eccentricities of all the planets, that is, why the physical orbits of the planets are the way they are. So, why the small eccentricities? Because it is on our planets that the hard and soft sixths depend, and thus upon which the crux of the whole musical system hangs. After working out how hard and soft harmony is distributed throughout to form one harmonic system Kepler said :

“Therefore, you have here the reasons, for the disagreements over very small intervals, smaller in fact than all the melodic intervals.”⁶⁷

The region of most importance for the harmony of the whole Solar System,⁶⁸ that between Earth’s Aphelion and Venus’ perihelion, forms Harmony in octaves with the outermost parts of the Solar System. Saturn, the highest planet, is in harmony at Aphelion with the Earth at Aphelion forming $1/32$ (which is continuous doubling of the octave $1/2$), and Mercury, the innermost planet, is in harmony at Perihelion with Venus’ Perihelion forming $1/4$ (one doubling of an octave $1/2$). Here the whole system is seen to make a giant counterpoint, echoing each other in an octave.

Also, in these outer planets, perfect harmonies were found among the converging motions in the pairs of planets, but not in each individual planets motion’s, while in the inferior planets, the opposite was the case.

chain. And although Bernoulli found his own construction for the catenary: Leibniz’ was unique. Because of his passion to demonstrate the perfection with which the Creator created the universe, only he discovered the true concept of the substance, a construction which expressed such perfection, both in its beauty, and in its power; his construction captured the irony of the paradox of the physical action of the curve. The relationship between the substance and the sense perceptible physical curve, is only knowable to the mind in form of a higher transcendental, the geometry of the complex domain. Therefore, modern critics who shriek, “but why must we talk of a Creator in relation to the universe? Science has nothing to do with it!”, should well pay heed to these historical truths. For, like Cusa’s transcendental, the existence of the physical complex domain, upon which modern science depends, would never have been discovered without Leibniz’ knowledge and demonstration of “the best of all possible constructions”, in the image of the best all possible Creator’s.

⁶⁷ *Harmony of the World* Book V, Chapter 9, Proposition XLIV

⁶⁸ *Harmony of the World* Book V, Chapter 9, Proposition XIV

And as was said above Earth and Venus had two perfect harmonies $5/8$, and $3/5$ between their extreme motions, so that they change the kind of harmony either soft or hard, whereas between Mercury and Venus there are two perfect harmonies, but which do not change their kind of harmony. And as Venus is the most imperfect in its own proportions and the smallest eccentricity, so Mercury is the most perfect forming a perfect $5/12$ and the largest.

In conclusion, Kepler showed that the physics of the system, that is the orbital elements of each planet, occur as a secondary product to the musicality of their motions, which in turn itself, is secondary to the idea of the Great Composer. Physics is an afterthought to the principle of perfection and reason. An intention to create a harmonic organization of the system as a whole generated each particular harmonic proportion, and as a consequence, each particular physical characteristic. Kepler then went on to derive all the orbital elements as shadows of the harmonies.⁶⁹

In demonstrating that the physics of the entire Solar System could only be known through the harmonies, how does that transform the definition of humanity as a whole?

Wrestle with this question: how can it be that the solar organization of everything is based on the same harmonic ratios that human beings created music with before we even knew this?

Look at the harmonics in human music. In the human organism, we can use our reason, our intellectual inquiry, to detect the relations of the sounds we make with our vocal chords to create pleasing tones. Those are instinctual if the ear and mind are trained to focus on certain properties of the voice. The harmonies are then organized to express even more. And as Kepler showed, when we turn our ears, our inner ears, to the heavens, we detect an ordered development which is the same way human beings communicate ideas in music. Thus, not only are we tuning ourselves to the universe when we sing, we then tune to the principles, and compose with them, imitating what only the Composer does.

And if music is nothing other than harmony detected by the human ear then the same harmonic organization, the same geometrical proportion exists in the small and in the large, in fact, in all physical principles. Therefore, as Kepler 'listened' to the Solar System to determine its characteristics, all these ratios can be examined with the 'inner ears' first to see if they are the correct ones. If they are harmonic, then the organization is true, if not, then not true. What area of physical science is not affected by this discovery?

Such was Kepler's revolution. He demonstrated all of the indicated paradoxes of an 'imprecise' continuously changing universe that Cusa had indicated, applied Cusa's investigations into the infinitely small and large. But Kepler having demonstrated all of the implications of Cusa's physics, went further, to change the universe as a whole, in redefining it's 'imprecision' as only knowable, through measurements with the same proportions—the ones Kepler most prominently

⁶⁹ <http://wlym.com/~animations/harmonies/site.php?goto=proposition48.html>

derived from Cusa's conceptions—found within the human soul. Therefore, the human soul is shown in the organization of the entire solar system, as a universal principle.

And that is '*real fun*'.

“Marvelous is this work of God, in which the discriminative power ascends stepwise from the center of the senses up to the supreme intellectual nature...in which the ligaments of the most subtle corporeal spirit are constantly illuminated and simplified, on account of the victory of the power of the soul, until one reaches the inner cell of rational power, as if by way of the brook to the unbounded sea, where we conjecture there are choirs of knowledge, intelligence, and the simplest intellectuality.”

“Since the unity of humanity is contracted in a human way, it seems to enfold everything according to the nature of this contraction. For the power of its unity embraces the universe and encloses it inside the boundaries of its region, such that nothing of all of its potentiality escapes.... Man is indeed god, but not absolutely, since he is man; he is therefore a human god. Man is also the world, but not everything contractedly, since he is man. Man is therefore a microcosm or a human world.”

—Nicolas of Cusa, *On Conjectures*

APPENDIX: Cusa on the Human Soul

There are four elements of the soul, the intellect, the rationality, the imagination, and the senses. The rationality is aroused by the senses, which in turn arouses the intellect.

Cusa relates the capacity of each part of the soul through a metaphor of a sphere.



When the senses perceive a sphere, only the part of the sphere seen by the eyes, or touched by the hands, is real, therefore, no sphere actually exists for the senses. But for the imagination, a round sphere is conceived, even though the eyes only see a part of it. The imagination has the power to conceive all parts of the sphere, thus making it whole. Further, the rational soul understands the sphere in its rational form, as equal radii from the center in all directions. But the intellect conceives of a sphere, which is infinite, with the center coinciding with the circumference. Cusa says, that the true sphere is the one the intellect perceives. I intellect perceives the potentiality of all spheres of all sizes, and even a sphere beyond all sphere's. As if in a flash, the mind perceives a point becoming an infinite sphere, the sphere of all spheres. Likewise with the circle, the rational concept of it, is not the true one, if it is merely that which all lines to the center are equal. The true circle in absolute unity is without lines and circumference. The true circle is infinite, which is infinite straightness, and serves as a measure for all things.

See Sphere Animations:

Sensible: <http://wlym.com/~animations/ceres/PDF/Michael/sensiblesphere.swf>

Imaginative: <http://wlym.com/~animations/ceres/PDF/Michael/imaginativesphere.swf>
Rational: <http://wlym.com/~animations/ceres/PDF/Michael/rationalsphere.swf>
Intellectual <http://wlym.com/~animations/ceres/PDF/Michael/Intellectual%20Sphere.swf>

The intellect depicts the sense perceptible in the imagination. The imaginative representation is then enfolded by the rationality into a unity of knowledge. It unites the otherness of the senses in the imagination,, and then unites the otherness of the imagination in the rationality, and lastly, the intellect enfolds the varied otherness of the rationality into the unity of itself. Likewise, the intellect becomes actual through the descent to the senses. The unity of the intellect descends to the otherness of rationality, and the unity of the rationality descends to the otherness of the imagination and so on.

The intention of the intellect is to become actual. In that way, Man submits himself to the senses in order to attain understanding. He says our intelligence is like a spark of fire concealed under green wood, which needs the senses to draw forth the heat in the wood. The more powerful is the actuality of fire, the more rapidly it causes the ignitable to become actual. And as the imagination needs the rationality to be intelligible, so colors need light to be seen, as one's vision cannot move directly to color without light.

Ascend higher therefore: the rationality is conveyed into the intellect through itself, as light is into vision, and the intellect descends through itself into rationality, as the vision proceeds to light. Now all things are defined by that which measures it, and so the rationality is defined by, and is the intellect descending into it.

Although the rationality partakes in the otherness of the senses, the intellect is the unity of the rationality, and thus precedes otherness. Cusa says, that the rational higher nature, which also absorbs the unity of imagination, and which is concealed in the light of the immortal intellect, is also immortal, like light that cannot be obscured.

Therefore, the difference between men and the beasts is that human rationality is absorbed in the immortality of the intellect. It is always intelligible through itself light as light is visible through itself. Animals have an otherness of rationality, like the otherness of colors which are not visible through themselves.

The absolute intellect embraces truths that have been unified by the rationality. Taking the origin of truth from sensible things is not absolute knowledge. But, if the otherness of the senses enfold into a unity in the rationality of the soul, and all of the different rational operations enfold into a unity in the intellect, what is the intellect an otherness of, in which it is enfolded as a unity?

Cusa says, the intellect is the otherness of the infinite Unity. And so, although the intellect can never attain infinite unity, it moves as far from otherness as possible to attain the highest unity. The perfection of the intellect is its continual ascension toward the infinite cause of all causes.

Without the rational soul, then time, the measure of motion could neither be, nor be known, since the rational soul is the measuring scale of motion, or the numerical scale of motion. And conceptual things are created by Man, as things existent by god. Soul creates instruments to discern

and know. They unfolded their conceptions in perceptible material. And man creates instruments like temporal measures. Since time is the measure of motion, it is the instrument of the measuring soul. Therefore, the soul's measuring does not depend on time, rather the scale for measuring motion, time, depends on the soul. As the eye and sight, eye is the instrument of sight the rational soul does not measure motion without time, but the soul is not subjected to time. We are not the slaves of our instruments. Thus, the soul's movement of distinguishing cannot be measured by time, souls' movement cannot come to end at some time, and thus its movement is perpetual. And its nature is not corruptible as all things subject to motion dissolve, but rather, the soul measures motion with time; therefore, that which measures motion, is the form of motion and is not subject to motion in that way. Thus, the soul is not corruptible in motion, nor is subject to time. Thus it is eternal, and immortal.