GGA April 14, 1757

Herr Pr. Kästner announced in his lecture a document of 18 quarto pages, [published?] byPockwitz and Barmeier, indicated by the following title: *Formulam Cardani, aequationum cubicarum radices omnes tenere*. The cubic roots by means of which a cubic equation may be solved according to Cardan's formula become, as is known, impossible, if the equation has three possible roots. This is a curious thing for all algebraists: Newton said that the expression of the root becomes impossible, because all three would have to be expressed at once, and that this could not happen. Stübner considered this a use of the principle [Satz] of sufficient reason. Kästner however noted other mathematical investigations, where the principle of sufficient reason is actually viable; here, the application is incorrect, because exactly the same thing can be said of impossible roots; and really/in general[überhaupt] the thing which it is desired to explain does not exist, but rather all three possible roots are also stated at once through Cardan's expression . [Darauf] he related the history of the discovery of this rule from Cardan's algebra. Scipio Ferreus discovered it, and taught it to Antonio Maria Floridus. Nicholas Tartaglia was embroiled in a dispute with him, and, in order not to be outdone, discovered the same rule for himself, and after many requests shared it with Cardan, although with the proof omitted. Cardan discovered the proof, and saw [considered? gesehen] that more, similar rules could be discovered, because up to that point every change in sign [power? Zeichen] had a new rule. Kaestner cited the relevant portion from Cardan, and explained the notation of the Cossists, of which Cardan made use, naming various [people] who had made use of [vorgetragen] these same rules, and who mostly taught that the case of all possible roots would not be contained in them. Clairaut correctly noted in his Algebra, that from the fact that two impossible cubic roots could be extracted, it did not follow that the sum of these cubic roots would be likewise impossible: to the contrary, he transformed every cubic root into an infinite series with/like Herr Nicole and showed that in both sums the imaginary parts of this series cancel each other out. This method of using infinite series was already shown by Leibniz in a letter of 1698 which is in part III of Wallis' works, where he noted that the impossible magnitudes in the sum would *virtualiter*--as he worded it--cancel, so, Herr Nicole did not, as he purported, teach this first. Kaestner admonished [erinnert] however that the conclusion is not correct, that if a series contains purely possible terms, it also expresses a possible magnitude: thus $\sqrt{(1-u)}$ contains only possible parts, even if u is larger than 1; although the series therefore does not converge, which [welches] is however no sign of impossibility. That is to say, in such cases the completion [Ergänzung], which should come to every series, to express the magnitude exactly and completely, becomes impossible, and thus Herr Nicole would have needed to show that these impossible completions [Ergänzungen] cancel in the sum. In an essay contained in the papers of the Royal Prussian Academy of Sciences, Herr König attempted to explain the impossibility of a formula of Cardan's type for the case of all possible roots, completely via logic[al argument]. After the relating what several others had done, Herr Kaestner displayed/indicated his intention, to so clarify this apparent inconsistency/puzzle, that the methods taught by Newton, Clairaut and Maclaurin et al., to extract the cubic root from the sum or difference of a rational and an irrational number, would be unnecessary, because these methods do not always occur [apply? stattfinden]. He thus proved without qualification [ohne einige Einschränkung], if A, P, Q, signify possible magnitudes, and B an impossible quadratic, then $\sqrt[3]{(A \pm B)}$ would be $= P \pm Q\sqrt{-1}$ where P, Q, may be irrational.

From that he derived/concluded that Cardan's formula gives three possible values if *B* is impossible, but two impossible values whenever *B* is possible. This is based on [gründet sich] the two impossible roots which every cube has besides its possible one, and completely explains everything that appears in Cardan's formula. Kästner noted further that with the extraction of a cube root to the [zum] usages [Gebrauche] of this formula a circular argument is actually committed, and a cubic equation is solved, whose roots are the halves of the roots of that equation which it is desired to solve by this means, and which could have been solved just as easily. Thus Maria Gaetan Agnesi's sentence from her *instituzioni analitiche* is verified, which said that this rule was of very little use. Kaestner also showed how to find Cardan's formula by Landen's method (see the GGA of this year, pages 164, 165), and indeed through a shorter calculation, because he threw out the second term, [and] similary how the solution is derived from the differential formula by the division of an angle into three parts [?], which, on account of his subtle [scharsinnigen] application of the infinitessimal calculus, desired to show whether he did not lie, that it could be had more quickly in another way.