

Göttingen Notices of Learned Matters
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A short treatise with the subject, *Geometrical Construction of the regular Heptadecagon*, is being submitted to the Royal Society by Herr ERCHINGER of Thuningen, in the Kingdom of Württemberg. The general theory of regular polygons obtains an enlargement and a new form by the well-known intimate union which it has been brought into with Higher Arithmetic - most especially in the smaller part of the theory of polygons: those which permit of themselves geometrical construction. Since the age of the Greeks, it was known that the triangle, pentagon, pentakaedecagon, and the polygons which arise from any doubling of the number of sides of those, each have that property [of constructability], and it was since believed (as well as assertively contended) that these were so, exclusively. Higher Arithmetic has learned that this was a mistake: as it lays open the true sources of the entire general theory, flows forth that innumerable other polygons besides those aforementioned are given which could become constructed geometrically of these, the heptadecagon is the most simple. This Analysis [(Higher Arithmetic)] embraces with equal ease the most general [concepts/problems] as well as the particular. Geometry [, however,] must always remain in the particular, where its progress from the simpler case to the more compound is halted by constantly increasing [tangles of] complexity, and it would only ever reach the [now] known next case with difficulty. Thus, the superiority of Analysis over Geometry shows itself in the clearest light. Nevertheless, it is always important, interesting, and desirable that the pure geometrical treatments become continually cultivated, and that Geometry should at least assimilate itself as a part of the new field which Analysis conquers.

The report is not well-known, that someone has publicly treated the construction of the heptadecagon outside of Herr PAUKER in the writings of the Kurdländischen Gesellschaft and his *Geometrie*. Hrn. ERICHLINGER accomplished it in another manner, more in the pure geometrical spirit, which consists in the following.

(The referenced figure, a straight line upon which the succession of Points *DBGAIFCE* lay, can each signify itself [?]) As one might please, produce an

assumed straight line AB forwards and backwards toward C and D , such that $AC \cdot BC = AD \cdot BD = 4 \cdot AB \cdot AB$; further, determine the points E, G upon the side of the produced line CA , such that $AE \cdot EC = AG \cdot CG = AB \cdot AB$, and the point F from the side A of the produced line BA such that $AF \cdot DF = AB \cdot AB$; finally, cut AE in I , such that $AI \cdot EI = AB \cdot AF$, where AI is the smaller part and EI the greater of AE . Then construct a triangle in which two sides are each $= AB$, the third being $= AI$. Describe a circle about this triangle; AI will then be the side of the regular heptadecagon inside the describing circle.

If one examines the correctness of this construction by comparison with that in the *Disquisitiones Arithmeticae Art. 354*, as one example of an established theory of the heptadecagon, then one easily notices that it is not other than a geometrical translation of that equation, out of which the application of the general theory led: in the action [above], the distances of the points C, D, E, F, G, I from A are not other than the magnitudes which the cited place signifies as $(8, 1), (8, 3), (4, 1), (4, 3), (4, 9), (2, 1)$, if one expresses the positive and negative signs by the placement, and the distance of the point B from A as set precisely in the direction taken $= -1$. However, the true merit of the treatise of Hrn. ERCHINGER is not so properly placed in the formulation of the construction itself, for there Analysis had already drawn out the simplest method, as it is in the correctness of his pure geometrical foundation, and this is accomplished with such exemplary laborious care - never straying from the pure elementarity - that it proves the character of the author and occasions the wish that all might find an awakening through his extraordinary mathematical talent.