Kästner's Lecture on Polygonal Solids

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Professor Kstners Lecture in the Royal Society d. Sciences on the 24 of May dealt with polygonal solids, which according to known laws are irregular, *(de corporibus polyedris data lege irregularibus)*, Mr. Eulers construction of regular solids by spherical trigonometry, and the measure of a solid angle, created in professor Kstner the idea, of applying this operation to irregular solids , where one immediately anticipates, that it will be harder, since the face not regular, also perhaps not all solid angles are the same e.t.c.

The beginning of the treatise consisted of some history of measuring solid-like angles, especially therein what Johann Broscius achieved. (One s. GGA 35 and 39 St.) Of the authors who have investigated irregular polyhedral bodies, Professor Kstner named Franciscan Flussatem Candallam, in Book 17 and 18 of his edition of Euclids Elements, Paris 1602. Kepler in the Harmony of the World where in Book II the solids are enumerated and represented, though without fully treating their theory. Professor Marpurg has, in his elements of progressional calculus, very properly represented the division of these solids, and taught clearly and with detail the fabrication of their nets, though without proof, and he himself desires thereof, that mathematicians would want to develop it more.

As these investigations do not belong in the elements, and their difficulty results in no benefit in common life, they have thus been somewhat neglected, the more so since the analysis, Cartesian as well as the Higher, was applied mainly to curved lines and curved surfaces.

Euler himself has made their application to planar and spherical trigonometrie, quite convenient, that he first published in the analytical Elementa Soliorum. The beginning of the essay, consisted of lemmas, among other things: the comparison of the area of a polygon, which is contained on the surface of the sphere between the arcs of great-circles, with the spherical surface, a theorem already taught by Broscious. (In the Gelerten Anziegen Page 339 S. Corrolary 16. Read Polygon instead of Tertragon.) It was demonstrated, how easily the regular solids can be derived from it, illustrated with the example of the Dodecahedron, the most difficult.

The first bodies considered by Professor Kaestnerare of the type that is enclosed only in the Rhombi. Conceivably such solid angles can be enclosed by no more than three obtuse angles of Rhombi, while four or five acute angles can stand about a solid angle, no more, since from the assumption that around the solid the Rhombi are to be identical, then their angles be counted. Four acute angles about one solid angle and three obtuse about another, give twelve Rhombi all around the solid; five acute make twenty Rhombi. One can designate these two bodies, with Professor Marpurg, the Rhombic Dodecahedron or Triacontahedron. The first has six solid angles enclosed by acute angles, and eight within obtuse ones; the second has twelve solid angles enclosed within acute angles, twenty within obtuse.

Each solid vertex which is enclosed by acute plane angles is located on the surface of a sphere, and those enclosed by obtuse angles in another sphere which is concentric with the other.

Since these bodies are enclosed in figures which are al of the same type, it is thus a test of a body which is enclosed in many figures, if a vertex is enclosed in two square angles and two angles of an equilateral triangle, the triangles each lie between a pair of squares, and have the same side as them. Thus we obtain all around six squares and eight triangles, the solid can be described in a sphere, whose radius is equal to the side of the square, has 12 vertices, Kepler called it: *Tessareskaidecaedron* oder Cubi Octahedron.

With this last operation, plane angles are set together around a vertex, and it is sought whether these figures tiled will enclose a solid. The figures are thus assumed, and used to bound a physical space. To this pertains spherical calculations and the determinations of physical angles, however it must be known, whether certain vertices, or the spherical triangles corresponding to them, exactly measure the spherical surface. The number according to which the measurement occurs, is the number faces of the solid.

It appears that the original discoverers of such solids arrived at them in another way, beginning with existing bodies and forming the others by cutting of pieces, the way a statue is formed from stone or wood. Thus Candalla cut a triangular pyramid from each corner of a cube, and did the same for all twenty vertices of the dodecahedron. These then give the solids, which are known as the octahedron and icosihedron. In great detail, he discussed their qualities, ratios, and how they become inscribed in the sphere and in other solids.

Professor Kaestner showed in the last section of his treatment what the nature of these two bodies depends upon. The first is the aforementioned, Keplers *tessarakaedecahedron*, and can thus show the difference of the two methods: of finding a solid from given sides, or from making cuts off of another.

More examples of such solids, and their general classification, cannot be addressed in the present treatise on account of their great extent and could perhaps have become made up for in the future.