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Hrn. Hofr.[1] Kaestner's lecture in the Royal Society dealt with: *De corporibus regularibus abscissis et elevates*[2], the 17th of May. The professor has, in many treatises, examined the various forms of geometric solids, mainly as they are determined to be of two or three classes by the regular plane figures which enclose [them], as well as [examining] several times what results when one cuts pieces from the regular solids by means of planes perpendicular to the straight lines from the center to the vertices of the solid angle: *de polyedris data lege irregularibus* diss. 1. Prop. VII. and *de sectionibus solidorum, crystallorum structuram illustrantibus*, both in *Comment. Math. ad* 1783,84.[3] In the latter he investigates in particular what cubes and octahedrons produce. He now examines the three remaining solids. Lemmas are: if from regular triangles, pentagons, or any regular figures in general whose endpoints are diminished by straight lines, equal segments along their sides are taken from every angle's vertex. As long as each of these segments is smaller than half the side, one can say it will be taken away on each end of the side, just as triangles are taken away from the figure by those lines which subtend the angle. In this way, figures are formed with twice as many sides as had the figure from which these were taken, whose sides are alternately equal, and make equally large angles with one another: semi-regular polygons; they become entirely regular by certain ratios of the segments to the side in the Collected Geometric Treatises I. collection 46th treatise.[4]

If the segment is half of the side, then a polygon is formed, similar to the one given, [and] described in it. If a segment greater than half of the side is taken, one can certainly not say that it is permitted to be taken away from the side twice, but the endpoints still permit themselves to be connected with

[1] 1A title bestowed upon academic officials indicating honorable service. Originated with the Holy Roman Empire of the German Nation as a title for a representative to the 'Hofrat'- a council/governing body for particular regions. There is really no equivalent in English, but 'Sir Councilor' or 'the Right Honorable,' serve to give a taste of this appellation.

[2] 2Latin: On truncated and stellated regular solids. An extensive article of the same name appears in the *Commentationes Mathematicae* on the same date. The full article can be found here: http://dz-srv1.sub.uni-goettingen.de/sub/digbib/loader?did=D233708

[3] 3 Commentaries on Mathematics, 1783/84. This is a section of a yearly Latin publication at Goettingen, *Commentationes Societatis Regiae Scientiarum Gottingensis recentiores*, to which Kaestner, as well as other notable professors such as Gesner, Mayer, Heyne, and later Gauss, regularly submitted articles.

[4] *Der Versammelte Abhandlung: Geometrische Abhandlungen*

straight lines as before, and thus form a figure inside the one given, similar and concentric to it, and as greater segments are taken, this figure decreases, and eventually comes together in the common center. The equilateral triangle thus produces first a semi-regular hexagon, which becomes regular if the segment is one third of the side, and is transformed into a triangle described in the one given if the segment of the side is one half, and were the segment greater still, smaller triangles would thus be formed, which will come together in the center when the segment is two-thirds of the side. If one takes the segment greater still, the triangle is thus restored to its former condition, in a condition opposite to its previous one, and increased; the segment taken equal to the whole side produces a triangle that corresponds to the one given.

Now for the solids. The sides of the solids are always called **a**; one calculates equal segments $=\mathbf{x}$ from the vertex of each solid angle along each of the sides which enclose them. Planes laid through their endpoints remove pyramids, whose vertices were the solid angle, and cut the faces of the solid (*hedras*). The cuts of the faces produce figures, as was established by the foregoing lemmas. These figures and the bases of the aforementioned pyramids enclose the remnants of solids, the truncations (*truncus*). This procedure, to begin from the vertex of a solid, differs from that which was required in the treatise on sections of solids:5 [to begin] from the outside inwards, and from the inside outwards, as in drawings of fortresses.