Part Three

Short and Easy Method, to find the Approximate Partial Determination of the Path of a Comet

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33.

Thus, from the previous is proven that if one gradually guesses the path of a comet through innumerable trials, not with `de la Caille`, I can virtually say, [it] will not necessarily be an entirely true [one], the hypothesis must only be assumed to be coming nearer the truth, which simplifies this entirely too complex problem of a first approximate solution. With Hrn. Boscovich the part of the path between observations were assumed as traversing a strait line and with constant velocity, [an assumption that] is somewhat too risky and gives in most cases a determination still too deviant from the truth, since here, not one, but rather two false hypotheses are made: straight line motion and constant velocity. The truth would be much more closely approached if one merely satisfied the theorem that the chord of the path were cut by the middle *radius vector* in proportion to the times. And by only assumeing likewise the chord of the path of the Earth is also cut in proportion, then an indeed indirect, but easy and convenient method is obtained, with which to calculate approximate elements of a path of a comet, than one might hardly imagine, considering the difficulty of the problem.

34.

Fig. 1.

Thus if S is the sun, A, B, C three positions of the comet with the consideration that the intervening times between the three is not very different [large?] and generally remote observations not far from each other, a, b, c the three positions of the Earth [corresponding] to the times of the three observations: then I assume that the middle *radii vectors* SB, and Sb cut the chords AC, and ac at D and d in proportion to the intervening times, so that if the time between the first and second observations is taken to be t', between the second and third observation t", ad:dc = AD:DC = t':t". This presumption is not entirely true: however, it deviates very little from the truth if the arcs AC and ac are small. The times in actuality behave as the parabola and ellipse sectors ANBS, BMCS, anbS, bmcS: the section of the chords however, as the triangular sectors

ABS, CBS, abS, bcS. Except 1) if the arcs are small, generally the parabolic and elliptical sectors not much greater than the triangular, that is, only by the small segment $ANBA$, *anba*, BCB, *bmcb*; it is clear that [is true] if the arcs and thus also the sectors themselves are small magnitudes of the first order; 2) the segement will be greater or smaller than the sectors, only admittedly not in simple proportion to the sectors; and 3)there is for every parabolic and elliptical arc a *radius vector*, which sufficiently cuts the chord in proportion to the times, or also again, the small segment $ANBA$, $BMCB$ etc. are exactly of the proportion AD:DC. Newton, Gregory and the excellent Lambert have investigated and generally shown under which circumstances this occurs with the parabola, that with small arcs very little errs from this proportion, if the times are not very disparate. With the path of the Earth, the error, in the case of the almost equal intervening times, is, if anything, just as minisucule, since this path varys little from a circle.

35.

According to this premise, [the proportion of] the apparent positions of the comet to the time of the middle observation, which it would have, if the Earth had stood in d and the comet in D, can now easily be determined. Since firstly, the apparent positions of A, D, C seen from a, d, c lie in a great circle of the sphere; secondly b, d, S, D, B also lie in a plane, therfore all points of the line BS, seen from any point of the line bS in one and the same great circle. Thus, one may seek only the intersection point of these two great circles on the sphere, to find the position of the line dD. The first great circle will be determined by the observed positions of the comet in the first and third observations, the second determined similarly by the middle observations and the position of the Sun at that time. Now one takes

$$\cot\pi = \frac{\tan\beta'''}{\sin\left(\alpha''' - \alpha'\right)}\tan\beta' - \cot\left(\alpha''' - \alpha'\right)$$

π is then an arc, which gives the point removed by α' [from the point] where the great circle drawn through the two outermost positions of the comet intercepts the ecliptic, namely by an angle η, which will be determined by the equation

$$\tan\eta = \frac{\tan\beta'}{\sin\pi}.$$

The longitude of the points, where the other great circle intersects the ecliptic, is $= A''$, or equal to the longitude of the sun in the middle observation, and its declination θ is found by

$$\tan\gamma'' = \tan\eta\sin\sigma.$$

36.

Fig. 2.

Since according to our hypothesis, the chord of the path of the comet AC, and the chord of the Earth orbit ac will cut the lines of sight Aa, dD, cC in proportion to the times, then namely this proportion must also occur with all orthographic projections of these chords and lines of sight. Thus if CDA is the chord or the path of the comet projected on the surface of the Earth orbit, acd, as before, the chord of the Earth orbit, [and] αA, dD, cC drawn according to the three given longitudes α', c'', α''', then

$$CO : AM = \frac{CD}{\sin COD} : \frac{AD}{\sin DMA} cO : aM = \frac{cd}{\sin COD} : \frac{ad}{\sin DMA}$$

Now, since

$$cd : da = CD : AD = t'' : t$$

and

$$Cc = CO + cOAa = AM + aM$$

then is obtained

$$Aa : Cc = \frac{t'}{\sin DMA} : \frac{t''}{\sin COD}$$

However, $DMA =$ the difference of the logitudes of the first and second observations $= c'' - \alpha'$, and $COD =$ the difference of the longitudes of the longitudes of the second and third observations $= \alpha'' - c''$: further Aa, Cc are the curtate distance of the comet from the Earth in the first and third observation, which we have named above ρ', ρ'''. According to this

$$\rho' : \rho''' = \frac{t'}{sin(c'' - \alpha')} : \frac{t''}{sin(\alpha''' - c'')}$$

thus

$$\rho''' = \rho' \frac{t'' sin(c'' - \alpha')}{t' sin(\alpha''' - c'')} = M\rho'$$

whereby the proportion of the curtate distance of the comet in the first and third observation is given.

37.

However, to find the value of M or the proportion of the curtate distance in this way, is neither generally practical, nor always the most convenient. There is namely 1)a case where it can not be used at all: that is, with the comet, whose apparent motion is almost orthogonal to the ecliptic, or whose longitudinal motion is very slight, [and] whose latitudal motion is considerable. Here, the arcs $c'' - \alpha'$, $\alpha''' - c''$ will be too small, and thus M will be found precariously. 2)One case where one must use it: namely with comets which move slowly in the vicinity of their quadrature, particularly with respect to the latitude. Here the following/consequent method can be awkward [?]; 3)one case, where it would be used with exceptional convenience: namely, if the intervening times are vary small, or the observations are not very exact. Here it will be permitted without misgivings, to use a'' instead of the corrected longitude c'', and thus spare yourself the entire calculation of 35. It is no less than if one supposes that the lines Bb, Dd (Fig. 2) are parallel to each other, and therefore can deviate[fehlen?] very little, if the arcs ac, AC are small, and thus the lines bd, BD are very small. Then one has presently

$$M = \frac{t'' \sin(\alpha'' - \alpha')}{t' \sin(\alpha''' - \alpha'')}$$

38.

Since all orthographic projections of the line of sights cut the chords in the said proportion, then, to find a more general practical formula, one may project these lines only on one plane, that stands normal to the ecliptic, and again, is also normal to the middle *radius vector* for the earth. As is generally known LAMBERT has also already with benefit [Vorteil] chosen this plane. Thereafter, one makes

$$\tan b' = \frac{\tan\beta'}{\sin(A'' - \alpha')},$$

$$\tan b'' = \frac{\tan\gamma''}{\sin(A'' - c'')},$$

$$\tan b''' = \frac{\tan\beta'''}{\sin(A'' - \alpha''')},$$

so that b', b'', b''' are the angles, which the line of sight in the projection makes with the chord projected on the Earth orbit. Now evidently, from this

$$\frac{\tan\gamma'}{sin(A'' - c'')} = \frac{\tan\beta''}{\sin(A'' - \alpha'')},$$

thus the calculation for the determination of c'' and γ'' will be unnecessary. If one sets now the projected distance[Abstand] of the first observation $= \delta$, in the third observation $= N\delta$, then, because here as well, the chord will be cut in proportion to the times,

$$N = \frac{t'' \sin(b'' - b')}{t' \sin(b''' - b'')}.$$

However, now

$$\rho' = \frac{\delta \cos b'}{\sin(A'' - \alpha')},$$

$$\rho' = M\rho' = N\frac{\delta \cos b'''}{\sin(A'' - \alpha''')},$$

consequently

$$\begin{aligned} M &= \frac{\cos b''' \sin(A''-\alpha') \sin(b''-b')t''}{\cos b' \sin(A''-\alpha''') \sin(b'''-b'')t'}, \\ &= \frac{\sin(A''-\alpha')(\tan b''-\tan b')t''}{\sin(A''-\alpha''')(\tan b'''-\tan b'')t'}, \\ &= \frac{(\tan\beta'' \sin(A''-\alpha')-\tan\beta' \sin(A''-\alpha''))t''}{(\tan\beta''' \sin(A''-\alpha'')-\tan\beta'' \sin(A''-\alpha'''))t'}. \end{aligned}$$

One very convenient expression for M, which can present a still somewhat more flexible calculation

$$M = \frac{(m \sin{(A'' - \alpha')} - \tan{\beta'})t''}{(\tan{\beta'''} - m \sin{(A'' - \alpha''')})t'},$$

by which one sets, namely on account of brevity

$$\frac{\tan{\beta''}}{\sin{(A'' - \alpha'')}} = m.$$

39.

Thus, from that, the proportion of the curtate distance of the comet to the earth of the first and third observations is given. Now the distance itself is to be found...

Figures.pdf

Fig. 1

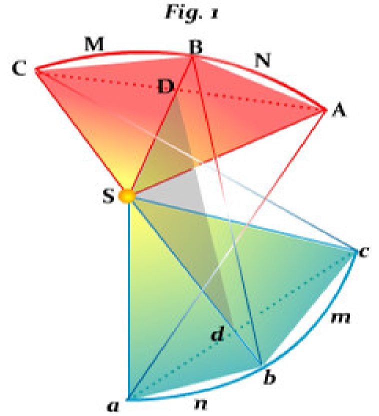

Fig. 2