On the presumed harmonic progression in the planetary intervals: as a supplement to the *Monatliche Correspondenz*, Nov. 1802, pg. 504

(Excerpted from a piece by Professor *Wurm*)

Blauteuern, December 27th, 1802.

It is no new remark, as Dr. *Gauss* seems to believe in the above-cited location, that the well-known [idea of the] progression in the planetary intervals is certainly not proven conclusively by that of Mercury. I have already expressly remonstrated that *it were only inapplicable to Mercury*, in 1787 in the *Berlin Astronomical Journal* of 1790 pg. 168, where this law has been more closely reviewed by me; I then already felt the unmistakable deficiency of a complete progression, and believed it to be impossible to hide from the reader. It is agreed that, in order to salvage this well-known law, one *must* assume that the progression begins first with Venus. Or one could also, for that matter, begin the progression with Mercury, expressing its distance by $3 + 1\frac{1}{2}$, and the following distances by 3+3, 3+6, 3+12, and so forth; as then one would have a series 0.45, 0.6, 0.9, 1.5, 2.7, 5.1, 9.9, 19.5, which indeed would always only approximate, but would not be fundamentally much less in accordance with the observed distances than the first above-mentioned law. A third law of progression, which, to be sure, ignores Mercury, and which is thus flawed with the same deficiencies as the first, could begin with successive locations from Venus: that is to say, setting its distance according to the observations as a foundation =0.72: thus the distance of the next planet to the Earth is =141/100 of Venus' distance, the distance of Mars is farther, =153/100 of the distance from the Earth, and the order afterwards is likewise, the following distances =165/100=177/100=189/100=201/100 of the aforementioned distances, if the distances are taken from the observations; the eccentricities of the planets are calculated with this third law, from Venus on: 0.72, 1.02, 1.53, 2.51, 4.90, 9.3, 19.17, the observations show 0.72, 1.00, 1.52, 2.77, 5.20, 9.54, 19.18.

One sees, that it is something easily done, to find out similar laws of progression by the dozens, if one is simply generous enough, and is satisfied with a rough approximation. However, should not this very fact -that a number of [these] presumed progressions could agree with the observation as well or as poorly as any other- present here most clearly the arbitrariness of the assumption, where, in general, laws and progressions appear in actual reality! To speak plainly, whatever the case may be, this is what it all amounts to in the end. *From Mercury to Mars, according to the observations, each successive interval is notably smaller than the double of the preceding distance; from Jupiter to*

Uranus each successive distance approaches nearly more than the double of the preceding. This, and only this, is an undeniable fact, and if one were to herald a yet [hidden*] Planet as nearly twice the distance as Mars is from the Sun, -not with bold conviction beforehand, but rather with humble surmise- between Mars and Jupiter (which constitute the sole exception, as they differ from one another by around more than the triple of the preceding distances), this simple observation alone could justify it; and yet, as an entirely new phenomena of our time discloses, how utterly ill-advised and how at the least ambiguous it is, to attach such a modest *Perhaps* to that fact,: for in the same interval, which the *Perhaps* denoted, instead of one hoped for Planet between Mars and Jupiter, the heavens have bestowed *Twin children* upon even those of us who had not expected it.

Everything of which we can see virtually no physical foundation that proceeds beyond the already mentioned fact, all such determined rules following from laws and basic progressions in the Planetary distances are nothing more than artificial, of which the archetype of Nature has taken no notice, and as soon as one adds to that so-called law a further system of conclusions, or wants to surpass it with observation: thus one begins, in astronomical subjects, not to *philosophize*, but rather to *dream*. Every one of these kinds Dreams are denied; but only inasmuch as they show nothing more, as is the case, than to not master nature, and to want to impose their petty considerations of an entirely too symmetrical model on the edifice of the world. You know how I myself determined it so well in my own thus far pertinent treatise in the *Astronomical Journal* 1791 pg. 188, as explained in the [A.G.E. II B. pg 466], and also in what I have written to you for many months, soon after the discovery of Pallas.[1] One may not repeat enough however, for the sake of certain people, who do not

[1]The location, whereupon Prof. *Wurm* here makes reference, reads thus: "*Whether projected observations shall show, that* Pallas *does not conform to the known progression of planetary intervals, is thus a matter of indifference to me. May so many of our systems and hypotheses fall to pieces by prompt and careful investigations into the heavens! 'Time destroys the fiction of opinions' [Latin]. If only the truth prevails (what should be of sole interest to us), if only our knowledge of Nature be ever more furthered through its experience!*" It is in fact high time, to censure the misuse and the foolery, which are pushed with these presumed progressions in the planetary intervals. Thus, for example, a particular piece in a very dear Journal (which even holds the *semi-major axis* and the *greatest distance* [aphelion] as identical)drivels such intolerable rubbish regarding a certain planet named *Ophion*, the next past *Uranus*. Another refers, in the Berliner astronomical journal of 1805, to *Wurm*'s formula for each law of progression, in order to draw out the consequences, which however do not *permit* themselves to be derived therefrom. Prof. *Wurm* writes to us thus: "*How greatly do I regret ever having written a syllable on this subject, when I did not wish to, since other good people also believed this astronomical jest [*Lusus ingenii-*(numbers game)], and that they themselves formerly have occupied themselves therewith... I wish to forestall it, so that in the future no one else may refer to a related work of mine in order to utilize it in an improper fashion. I demonstrate in my small piece the inadmissibility of these progressions, and prove, that there is no true single complete progression, consequently entirely nothing is suitable, than-to astronomical clockwork for the one, where not entirely to confusion of the mind for the other.*"

Of the same opinion is another good mathematical mind, who has traced the springs of the first idea of this clockwork. We thus permit thereto this pertinently imparted notice, from the Prof. *Vieth* of Dessau, to follow.

understand the *Jest* or do not wish to understand it, that -raving, and making reasonable conclusions, are two different things, that the gift of astronomical prophesy will only be obtained through rigorous and level-headed application of geometrical truths, and that everything, which is not based upon this, is a completely foreign trumpery to the science of Astronomy.

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Notice

In the *Monthly Correspondence*, the progression in the planetary intervals has come up several times since the discovery of *Ceres* and *Pallas*. In the June issue of 1801 it was *Bode*, attributed to the same *Titius* in the July issue of 1802. *Titius* says in the note to Bonnet, after he had brought up the progression (where incidentally a point instead of a ÷ always stands in between the numbers:) "Which noteworthy relationship and such contemplations, which Mr. *Bonnet* believes to be at first noted by Mr. *Lambert*, were put forward by the Baron *von Wolf* more than forty years before in his German Physics."

As I coincidentally happened upon *Wolf*'s writings, the place, that *Titius* presumably meant, fell into my hands. It stands in *Wolf*'s "reasonable thoughts from the viewpoint of natural things," fourth edition, Halle 1741. Chapter VIII "on the construction of the world," section 85 pg. 139 and makes out thus: "The planets, which move about the sun, stand very distant from one another. If one divides the distance of the Earth from the Sun in ten parts, the distance of Mercury from it thus comes to be 4; of Venus 7; of Mars 15; of Jupiter 52; of Saturn 95..."

"In such a conformation stand Mercury and Venus 3, Venus and the Earth also 3, Earth and Mars 5, Mars and Jupiter 37, Jupiter and Saturn 43 parts from one another."

Wolf does not say, whether he has discovered or borrowed this. Shame! that one could thus lose the genealogy of such an idea.

Wolf's numbers for Mars and Saturn incidentally deviate somewhat from the known progression, which are placed not at 15 and 95, but rather 16 and 100. However, it also does not seem, that *Wolf* had actually intended on a progression following the law 4; 4 ÷ 3; 4 ÷ 2.3 and so forth. He also finds in the interval between Saturn and Jupiter nothing in particular, in contrast much more in that between Mars and the Earth where he says it pg. 140: "It seems to be contrary to the distance between Mars and the Earth. For the space between both is greater, than between the Earth and Venus, which is the same with the space between Venus and Mercury, and nonetheless Mars is much smaller than Venus."

He had namely beforehand of necessity spoken of relative mass distances because of satellites and eclipses.

The ease, which man finds in the laws and regularity and symmetry of the numbers and magnitudes, such a progression gains much approbation in our humanly minds; but nature in its great works does not comply accordingly,

and mostly operates upon ratios, which in our language are called irrational. One should not assume that the mentioned progression were proven true like *Kepler*'s laws. Who knows if still more such small planetary globes or asteroids do not float about, which could not be conformed to the harmonic series.