

# Fermat to Mersenne

December 1637

MY REVEREND FATHER,

1. In the letter of M. Descartes that you took the trouble of sending to me, I have seen the succinct responses that he made to the objections that I had raised against his *Dioptrics*, to which I would have responded much earlier if my necessary occupations had not prevented me from doing so, for which M. de Carcavi will vouch for me. I first must tell you that it is out of neither envy nor emulation that I continue this little dispute, but only to discover the truth; to which end I think M. Descartes will not be unwilling, all the more since I know his very eminent merit, of which I make you a very explicit declaration. I will add, before getting into the matter, that I do not want you to circulate my writing more broadly than if it were an intimate conversation, in which I place my confidence in you.

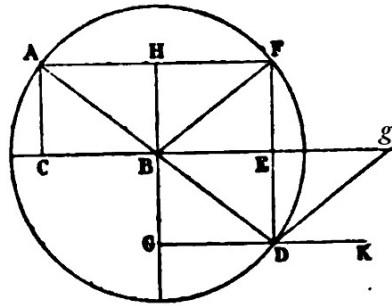
2. I make admirable brevity of our dispute, which dispute nevertheless I could make last longer, and prove that the author has accomodated his *medium*<sup>1</sup> to his conclusion, the truth of which he was previously certain of; for, when I insisted that his division of the components of motion was not made in the right manner, since we have an infinite number, I would reduce him to the proof of a proposition which was not easy for him. But, since we do not doubt that reflections are made at equal angles, it is superfluous to dispute the proof, since we know the truth; and, without arguing I deem it better to come to refraction, which is the goal of *Dioptrics*.

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<sup>1</sup>The pun is intended; Fermat means that Descartes both fit his means to his end, and treated the medium through which light moved accordingly.

3. I recognize, along with M. Descartes, that force or moving power differs from the determination, and, consequently, that the determination may change without the force changing, and likewise for the contrary.

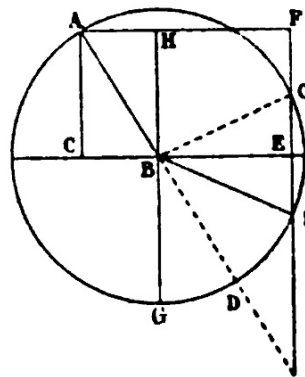
**Fig. 53.**



The example of the first case is seen in the figure on page 15 of the *Dioptrics*, where the ball directed from point A to point B (*fig. 53*) is deflected to point F, such that the determination to move along line AB changes, without diminishing or changing the force which continues its movement.

We can use the figure on page 17 for the second case (*fig. 56*). For, if we imagine that the ball has been directed from point H towards point B, and that it then falls perpendicularly upon the cloth CBE, it is clear that it will traverse it along line BG, and thus its motive force will be weakened and its movement retarded without a change in direction, since it continues its motion along the same line HBG.

**Fig. 56.**



4. I now return to the demonstration of refraction using the same figure on page 17.

*“Let us consider,”* says the author, *“that of the two parts of which one might imagine this determination is composed, it is only that which moves the ball from high to low that can be changed in some way by its encounter with the cloth, and that which causes it to move from the left towards the right must always remain the same as it has been, because the cloth is in no way opposed to motion in this direction.”*

5. First of all, I remark that the author has not remembered the difference that he had established between the determination and the moving force or the speed of motion. For it is quite true that the canvas CBE weakens the movement of the ball, but it does not prevent its motion from above to below, and, although it be slower than before, it cannot be said that, because the motion of the ball is weakened, that the determination to move from high to low has been changed. On the contrary, its determination to move along line BI is just as much composed, in the sense of the author, of that which causes it to go from high to low or that which causes it to go from left to right, as the first determination to move along line AB.

6. But let us concede that the determination towards BG, or, to speak like the author, the determination to go from high to low, may be changed: we can conclude from this that the determination towards BE, from left to right, is also changed. For, if the determination towards BG is changed, it is because in comparison with the original motion, the ball, now deflected and taking the path BI, now advances proportionally less towards BG than towards BE, we may also say that it moves in a further proportion towards BE than towards BG than it did before. If the first makes us understand that the determination towards BG has changed, then the second can cause us to consider that the determination towards BE has changed, since change is just as much caused by augmentation as by diminution.

7. But let us again concede that the determination of high to low may be changed, and not that of left to right, and let us examine the conclusion of the author, which we quote: *“Since the ball loses nothing of its determination to go towards the right, in twice the time that it has taken to go from line AC to line HB, it must go twice the distance towards this same side.”*

8. You see how he falls into the first pitfall, not distinguishing between determination and the force of motion; and to help you understand more

clearly, let us apply his reasoning in another case.

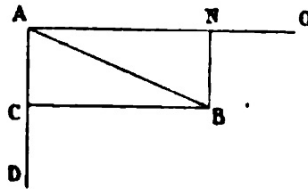
Let us suppose, in the same figure, that the ball be directed from point H towards point B. It is certain that it will continue its movement along line BG and that its determination will not change at all; but also its movement is slower along line BG than it was earlier, yet nevertheless, if the reasoning of the author is true, we could say:

Since the ball loses nothing of its determination of advancing towards HBG (since it is entirely the same), therefore, in the same time as before, it will traverse the same distance.

You see that this conclusion is absurd and that to make good of the argument, the ball would have to lose nothing of either its determination or its force, and hence you have a very manifest paralogism.

9. But, to entirely destroy the proposition, we must examine two sorts of compound movements which are made along two straight lines.

**Fig. 57.**



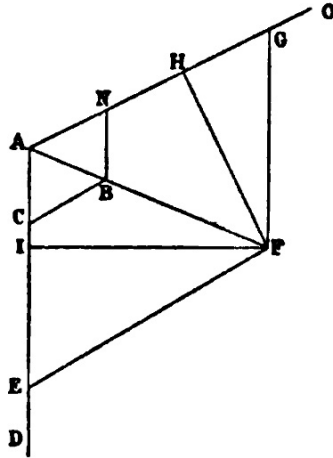
Let us consider, for example (*fig. 57*), the two lines DA and AO, which enclose angle DAO of whatever size you wish, and let us imagine a weight at point A, which descends along the line ACD in the same time that the line moves towards AN, with the condition that it always make the same angle with AO, and that point A of this line ACD be always on line AN. If the two motions, of line ACD towards AO, and the weight along the line ACD, are uniform as we may suppose them to be, it is certain that the compound motion will always direct the weight along a straight line, AB, along which if you take a point like B, from which you draw the lines BN and BC parallel to lines DA and AO, then when the weight will be at point B, it could, in an equal time, be at point C if it made only the movement along ACD, and at point N if it made only the other movement; and the proportion of the force which drives it along AD to the force which drives it along AO will be as AC is to AN, or as BN is to BC.

It is only this sort of compound movements that Archimedes and the other ancients used in the composition of their helices, of which the primary property is that the two moving forces do not hinder each other, remaining always the same. But, because this compound motion does not come so easily in practice, it must be considered in another way and a particular study of it must be made.

10. Let there be in the same figure a weight at point A, which is simultaneously acted on by two forces, one of which moves it towards AO and the other towards AD, and the first line of direction of the first movement is AO and that of the second is AD. If there were only the first force by itself the weight would always find itself on AO, and it would always be on AD if there were only the second force; but, since these two forces hinder and resist each other, let us suppose (and it must be remembered that we are assuming uniform motions here, for otherwise the compound motion would not be along straight lines) that in one minute of time, for example, the second force causes the weight to distance itself from its direction AO by length NB, which must be drawn parallel to AD; for the weight which is brought along AD by the second force, finding itself hindered by the first, will still be moved and will advance from A towards D along parallels to AD. Let us also suppose that in the same minute, the first force causes the weight to extend itself away from direction AD by a length CB parallel to AO for the preceding reason. It is completely certain that in one minute the weight will be at point B, which is the combination of the two lines BN and BC. The compound movement will thus be made along AB, and we can say that the weight traverses line AB in one minute.

11. Now let us change angle DAO, by making it larger, for example. In the following figure (*fig. 58*), making the same assumptions as before, I say that in one minute of time, the weight will move away from direction AO along line BN equal to the same BN in the previous figure. For, since the forces are the same, the second still hinders the determination of the first, and will cause the weight to move in its direction as before, in equal times, because there is always the same resistance. We will conclude the same thing of line BC.

Fig. 58.



The compound motion will thus be made along line AB, and line AB will be traversed as before in a minute of time; but since in the two triangles ANB in the first and second figures (*figs. 57, 58*), the sides AN and NB of the first figure are equal to those in the second, and the angles ANB which they enclose are unequal, it follows that the bases AB are unequal (and consequently that the compound movement will be less swift in the second than in the first), and that there will be such a proportion between the speed of the compound movement in the first figure to the speed of compound movement in the second as the length of line AB in the first to the length of AB in the second.

12. Now I take an arbitrary point F along the line AB, from which I draw lines FE, FG parallel to AO and AD.

$$FE \text{ is to } CB \text{ as } FA \text{ is to } AB, \text{ or } FG \text{ to } BN,$$

as the construction shows us: therefore

$$FE \text{ is to } FG \text{ as } CB \text{ is to } BN.$$

Yet, in the preceding figure, the lines BN and BC are respectively equal to lines BN and BC in the second figure, and we can, by the same reasoning, take a point along line AB in the first figure, to derive the same conclusion. Therefore, whatever point you take along line AB, let it be on the first or

the second figure, the parallels will be in the ratio of CB to BN, that is, it will always be the same proportion.

From point F, draw the perpendiculars FH, FI on lines AO and AD. In the parallelogram GAEF, angles AGF and AEF are equal since they are opposite angles: therefore the triangles GFH and EFI are similar, and consequently,

as EF is to FG, so is FI to FH

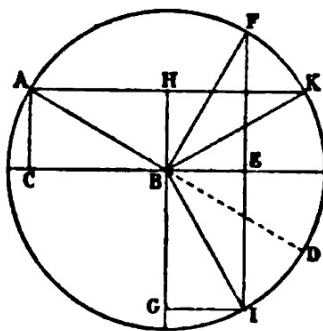
Yet

FI is to FH as the sine of angle DAF is to the sine of angle OAF,

and consequently, if you make the same construction on the first figure, you would conclude, to avoid prolixity, that the sine of angle DAB is to the sine of angle OAB in the first figure, as the sine of angle DAF is to the sine of angle OAF in the second figure.

13. Having assumed and demonstrated this, let us consider the figure on page 20 of the *Dioptrics* (*fig. 59*), in which the author assumes that the ball, having first been directed from A towards B, is pushed once more at point B by the racket CBE, which, in the sense of the author, doubtless impels it towards BG; such that from these two movements, one impelling towards BD and the other towards BG, a third is made which directs the ball along line BI.

**Fig. 59.**



14. Then let us imagine a second figure similar to this one, in which the force of the ball and that of the racket are the same, but angle DBG is larger in this second figure. It is certain, by the demonstrations that we have just

made, that there will be the same proportion between the sine of the angle GBI and the sine of the angle IBD, in the figure of the author, as that between the sines of angles GBI and IBD, in this second figure that we imagine, and which we omit here to avoid lengthiness. But, if the propositions of the author are true, there would be the same proportion between the sines of GBD and GBI in the figure of the author, as between the sines of angles GBD and GBI in the second figure that we have imagined. Yet, since this proportion is different from the other, it follows that it cannot hold.<sup>2</sup>

15. Moreover, the principle reasoning in the author's demonstration is founded on his belief that the compound movement along BI is always equally fast, although angle GBD, enclosed between the direction lines of two moving forces, has changed: which is false, as we have plainly demonstrated.

16. It is not only that I wish to affirm that in his manner of using the figure on page 20, my proportion rather than his must be used; for I am not of the mind that this compound movement should serve as the rule for refraction, on which subject I will tell you my thoughts at length at another time.

17. I will wait for the response to this letter, since you have given me reason to hope for it, and I will always be, my Reverend Father, your very humble servant.

Whatever I may write to you in the future, the excuse that you have noted at the beginning of my letter will still apply.

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<sup>2</sup>Thus Fermat concludes that if, following Descartes, we are to consider the movement of the refracted ray as the result of the movement of the incident ray and an action along the normal, the proportionality must exist not between the sines of the angles of incidence and refraction,  $\sin i$  and  $\sin r$ , but between  $\sin(i - r)$  and  $\sin r$ . To this end, he implicitly assumes a normal action independent of the incident motion. Descartes' hypothesis, on the contrary, is that the component parallel to the surface maintains the same value before and after the refraction. It is clear that one could not decide between these two assumptions *a priori*. (Note from the editors of the Œuvres.)