

Fermat to de la Chambre

August, 1657

Toulouse, August 1657

SIR,

1. I did not intend to obey you when you ordered me not to read the book that you wrote and sent to me.¹ The gift that you have made to me is a very precious mark of the friendship with which you honor me; but reading it caused me to consider the idea of this friendship as a good that deserves to be preserved with care, respect, and esteem. And to demonstrate it to you, I will not speak to you at all about your other speculations in Physics, even though they be full of very solid and penetrating reasoning; it will be enough for me to converse with you a little on the matter of reflection and refraction, even if this were merely to make amends by this letter for having lost a discussion which I addressed to you, several years ago already, on this same subject and which I knew had never come into your hands.² This is corroborated by the fact that I have now entered with you into a community of thought, and I even dare to assure you in advance that, if you would permit me to join a little bit of my geometry to your physics, we will accomplish a work of collaboration which will primarily put us in defense against M. Descartes and all his friends.

2. First, I recognize with you the truth of this principle: that nature always acts along the shortest paths. From this, you quite correctly deduce

¹*Light*, to His Eminence Cardinal Mazarin by Sir de la Chambre, counselor to the King in his Councils and his Ordinary Physician.

²It would be wonderful to have been able to see how Fermat's thoughts on light had matured over the intervening twenty years before receiving de la Chambre's book.

the equality of reflection and incidence, and that the objection of those who say that the two lines which sight or light traverse in a concave mirror are very often the longest, is nothing of great importance, if you only suppose, as another indisputable principle, that anything that presses against or holds firm upon a curve of any nature, can be considered to be pressing or holding firm upon a line tangent to the curve at the point of intersection: which can be proven by a physical reason aided by a geometrical one.

The principle of Physics is that nature performs its movements by the most simple paths. Now, since the straight line is simpler than the circular or any other curve, it is necessary to believe that the movement of a ray which falls on the curve is more related to the straight line which touches the curve than to the curve itself.

First, since this tangent line is simpler than the curve. Second (and this is what is borrowed from Geometry), because no line can fall between the curve and the tangent, by a principle of Euclid. So, thus, the movement is precisely the same on the tangent as it is on the curve to which the line is tangent.

And, supposing this, one can never say that the two lines which light or the ray trace out are sometimes the longest on concave mirrors, because even in this case they are found to be the shortest of all those which could be reflected off the straight line which is tangent to the curve. Consequently, it is neither necessary to suppose that nature acts under a constraint in this case, nor to conclude that it follows another manner of movement than that which it displays in the case of flat mirrors and all other types of mirrors. So, there you have your principle plainly established for reflection.

3. But, since it served us well for reflection, can we derive some use from it for refraction? It seems to me that the thing is easy and that a little geometry will allow us to tide over the difficulty or get out of trouble.

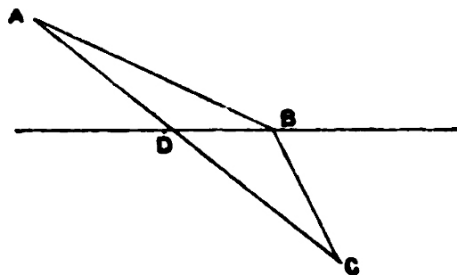
I will not bother myself with the refutation of the demonstration of M. Descartes. I contested it with him a long time ago when I spoke to him (in the words of Martial, *viventi atque sentienti*),³ but he never satisfied me. The use of these compound movements is a very delicate matter and one which must not be treated or employed without great precaution. I compare them to some of your remedies, which can be poisons if they are not well

³*when he was alive and sentient*, Martial, Epigrams., I, 11, 5. – see Fermat's letters of September and December, 1637.

and duly prepared. It should therefore be sufficient to say on this account that M. Descartes has proven nothing and that I am in agreement with you in rejecting his ideas.

But it is necessary to move on and find the reason for refraction in our common principle, which is that nature always acts along the shortest and easiest paths. It seems at first that the thing could not succeed and that you have yourself made an objection which appears invincible. For, since on page 315 of your Book (*fig. 87*), the two lines CB, BA which contain the angle of incidence and that of refraction, are longer than the straight line ADC which forms the base of triangle ABC, it follows that the ray from C to A, which goes through a shorter path than that of the two lines BC, BA, should, in accordance with our principle, be the sole and true route of nature, but which, nevertheless, is contrary to experience. But we can easily extricate ourselves from this difficulty by assuming, with you and with all those who have treated this matter, that the resistance of the media is different, and that there is always a certain ratio or proportion between those two resistances, when the two media are of a certain consistency and internally uniform.

Fig. 87.



Do not be shocked by what I say about resistance, after you have decided that the movement of light is made in an instant and that refraction is caused by nothing else but the natural antipathy which exists between light and matter. For, whether you grant me that the instantaneous movement of light can be contested and that your proof is not entirely demonstrative, or whether it is necessary to stand by your decision, *viz.*, that light avoids the abundance of matter which is its enemy, I find, even in this latter case, that, since light avoids matter and that one only avoids that which causes pain and resists, one can, without departing from your sentiment, establish resistance where you establish avoidance or aversion.

Let there be then, for example, in your figure the ray CB which changes media at point B, where it deflects to reach point A. If these two media were the same, the resistance to the passage of the ray along line CB would be to the resistance of the passage of the ray along line BA, as line CB is to line BA. For, were the media the same, the resistance to light's passage would be the same in each of them, and, consequently, the ratio of the traversed spaces would remain the same. From which it follows that, since the two media are different and consequently the resistance is different, one can no longer say that the resistance to the passage of the ray along the line CB will be to the resistance along line BA, as line CB is to line BA; but in this case the resistance along line CB will be to the resistance along BA, as CB is to another line whose ratio to the line BA will express the ratio between the different resistances.

Since: if the resistance through medium A is double the resistance through medium C, then the resistance along CB will be to the resistance along BA, as CB is to double BA. And, if the resistance through medium C is double the resistance through medium A, then the resistance along CB will be to the resistance along BA, as CB is to half of BA. So, in these two cases, the two resistances along CB and BA, being combined, will be expressible: either by CB combined with half of BA, or by CB joined to twice BA.

You see already without a doubt the conclusion of this reasoning: because, for example, let us take as given the two points C and A in two different media separated by the line BD, which would be of such a nature that the resistance of one would be double that of the other; it is necessary to find the point B where the ray going from C to A or from A to C, will be cut or bent.

If we suppose that point B has been found, and that nature acts always along the shortest and easiest path, the resistance along CB, added to the resistance along BA, will contain the sum of the two resistances, and this sum, in compliance with the principle, must be the least of all those sums which could be created at any point along line DB.⁴ So, in this case, these two combined resistances are represented, as we have proven: either by CB plus half BA, or by the same CB plus twice BA.

The question is thus reduced to this problem of geometry:

Given two points C and A and the line DB, to find a point on line DB for which, if you extend lines CB and BA, the sum of CB with half

⁴This is Fermat's first statement of his principle of refraction.

BA is the least of all similarly taken sums, or rather that the sum of CB and double BA be the least of all similarly taken sums;

and the point B which will be found by the construction of this problem will be the point where the refraction will take place.

You see from this that the ray must be cut and broken whenever the two media are different. For, even though the sum of the two lines CB and BA is always longer than the sum of the two lines CD and DA (or the entirety CA), nevertheless the line CB joined with half or double BA can be shorter than line CD joined with half or double DA.

I confess to you that this is not one of the easiest problems: but, since nature solves it in all refractions in order to not depart from its ordinary way of operating, why may we not tackle it?

I guarantee you in advance that I will find the solution whenever you want, and that I will even draw results from it that will solidly establish the truth of our opinion. I will first deduce: that the perpendicular ray is not bent at all; that light bends from the first surface without further changing the slope it has taken; that the bent ray sometimes approaches the perpendicular, and at other times departs from it, depending on whether it passes from a rarer to a denser medium or the contrary; and, in a word, that this opinion is exactly in agreement with all of the appearances. Such that, if it is not true, one can say, as Galileo did on another subject, that nature seems to have inspired us *per pigliarsi gioco di nostril ghiribizzi*.⁵

But I have failed in having forgotten that the subject of this letter was only meant to be a thank-you note. I pray you, Sir, to forgive its length, even if it were merely for the interest that you have shown in the subject, and to receive it in any case as a token both of the esteem which I have for your knowledge, and of the respect with which I am, sir,

Your very humble and affectionate servant,

FERMAT.

⁵by making fun of our whimsical ways.