Fermat to Clerselier

Sunday, March 10, 1658

Sir,

1. The conclusions that may be drawn from the proposition which serves as the foundation of the *Dioptrics* of M. Descartes are so beautiful and should naturally produce such beautiful effects among all of the works involving the art of refraction, that it were desirable, not only for the glory of our deceased friend, but much more for the augmentation and embellishment of the sciences, that this proposition be truthful and that it should have been legitimately demonstrated, and even more so because it belongs to those of which we can say that *multa sunt falsa probabiliora veris.*[1] I would like to go even further and compare it to that famous lie spoken of in Tasso, and which that poet assures us is more beautiful than the truth:[2]

Quando sarà il vero
Si bello, cho si possa a ti preporre?

I begin with that, Sir, in order to make you aware that I would be delighted if the dispute that I have had in the past with M. Descartes on this subject were ended to his advantage. I would be satisfied on two counts: the glory of a friend who I infinitely esteemed and who has for good reason been considered one of the great men of his time; the establishment of one of the most important physical truths; and the easy execution of the marvelous effects which would infallibly be deduced from it. For me, all of this would be worth much more than winning my case, even though I would count for nothing the

[1] There are so many plausible errors in truth.

[2] "When shall the truth be beautiful enough to be placed before you?" Jerusalem Delivered, Book II, Canto 22.

Mecum certasse feretur[3]

with which the friends of M. Descartes can still reasonably console his adversaries. I therefore put myself, Sir, in the position of a man who wants to be defeated; I strongly declare it:

Jamjam efficaci do manus scientiæ.[4]

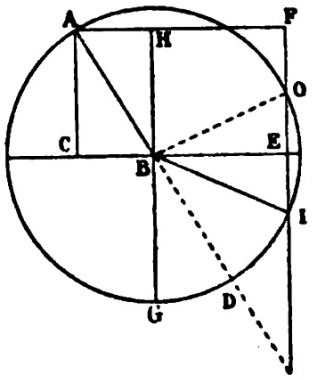
But, because demonstrations are forced reasons and, unless one is convinced by them, one could not be persuaded by them, let us see, Sir, if the agreement of the readers can escape the attention of our author [Descartes] and if we will be able to deal easily with the objections that appear to be in opposition to him. For that purpose, his demonstration must be followed word-for-word, and it will suffice for me to enclose in parentheses that which is not his but was added by me.

2. See therefore how he speaks at the end of page 16 in the French edition of his *Dioptrics:* "And first we will assume that a ball impelled from A towards B intersects at point B, not the surface of the earth, but a cloth CBE (*fig.* 56), which is so feeble and loosely woven that this ball has the force to break it and pass right through it, while losing only a part of its speed, one half for example. Now, under this hypothesis, in order to know which path it must follow, let us consider afresh that its movement differs entirely from its determination to move more in one direction rather than another, from which it follows that their quantities must be examined separately; and let us also consider that of the two parts of which we can imagine this determination is composed, it is only that which makes the ball move from high to low that can be changed in some fashion by the encounter with the cloth, and that by which it is made to move towards the right always remains the same as it has been, because the cloth is in no way opposed to motion in that direction."

[3]Ovid, *Metamorphosis*, XIII, 20.

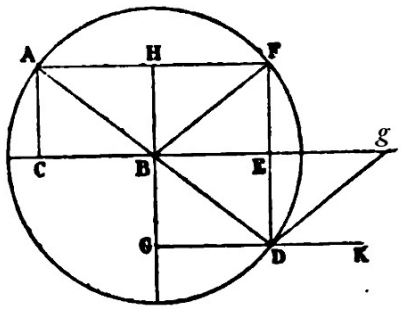
[4]Now, give an efficient hand to science. Horace, *Epodes*, XVII, 1

Fig. 56.



3. (But is this reasoning not somewhat contrary to common sense? Is the extension that he makes from reflection to refraction not somewhat forced? On page 13, [when discussing reflection] he assumed that the ball always goes at an equal speed, both in descending and in reascending, and that it continues its movement in a uniform medium; from this he deduces on page 15, that the encounter with the earth can very well hinder the determination that the ball had to descend from A towards CE, since it occupies all the space which is below C, but that it cannot at all prevent the other which causes it to advance towards the right, considering that it is not at all opposed to this direction; from which he infers the equality of angles of reflection and incidence.

Fig. 53.



But although his reasoning is true for reflection, some scrupulous skeptic would not hesitate to allege that there are three circumstances in refraction which must change the result, or at least serve as obstacles to accepting it without a new proof:

First, in the figure on page 17 or that on page 18 (*fig.* 56), the ball does not continue its movement with an equal speed, since, by the assumption, it loses, for example, half of its speed at point B.

Second, it does not always travel through the same medium, as it did in the figure on page 18.

And finally, the determination which causes it to go from high to low is not completely prevented by the encounter with the cloth or the water, but is only changed or diminished.

Now, it will be difficult for a mediocre logician to be able to agree that the result would be the same, despite the difference in these three circumstances. He will excuse his [[]]scrupulous logic, saying that he did not believe he was doing such violence to his principles when he acceded, in the figure on page 15(*fig.* 53), that the determination from left to right would remain the same, since a ball always moving at the same speed [as is the case for reflection,] could maintain one of its determinations when only the other was prevented; that in reflection the movement was made in the same medium; and finally, that since the determination from high to low was entirely prevented, he had no difficulty in consenting that the motion from left to right remains intact: as, when someone loses an eye, we say that the visual virtue is entirely conserved in the remaining eye.

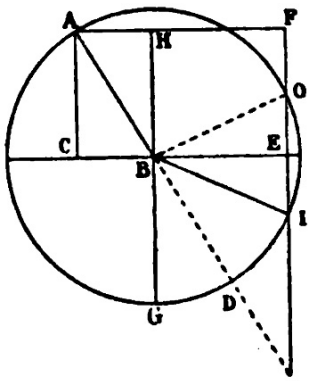
But, in refraction, everything is different. Would we like to obtain the consent of our skeptic without proof? Will the determination from left to right remain the same, when all the reasons which had persuaded him in the case of reflection have vanished? But that is not all: he has reason to suspect a misunderstanding, and, when he will have agreed that this determination to go from left to right remains the same, he has reason to suspect that the author will scold him on the explanation of that term. For, although he [Descartes] has protested that the determination is different from the power that moves, and that their quantities must be examined separately, if our skeptic concedes in this instance that this determination from left to right remains the same in refraction, that is to say that it conserves the same aim or direction, next it appears that the author would like to make him concede that the ball, whose determination towards the right has not changed at all, advances just as fast towards the right as before, although its speed and the medium through which it travels have changed.

But since it does not yet appear that such a violence is intended for him, he does not yet believe it is time to give up the respect he attaches to the name of M. Descartes, and he is willing to concede to him, solely on his

word, that this determination towards the right shall remain the same, as long as he does not raise the issue of the time it takes for the ball to move in that direction: because M. Descartes himself has confessed that the force which moves and the determination are two quantities which have nothing in common, and that they must be examined separately.)

4. "Then having described the circle AFD around center B, and having drawn at right angles to CBE the lines AC, HB, FE, such that there be twice as much distance between FE and HB as between HB and AC, we will see that this ball must tend towards point I. For, since it loses half its speed in passing through the cloth CBE, it must take twice as much time to pass below B to a point on the circumference of the circle AFD as it took above to pass from A to B: and, since it loses none of its determination to move towards the right, in twice the time that it took to pass from line AC to line HB, it must make twice as much headway towards this side."

Fig. 56.



5. (Here is the ambush, and the too-great credulity of him who had overcome all his scruples on the first article, here receives a new attack. The author has reason to hope that since our skeptic has already conceded that the determination towards the right will remain the same, he must not now retract it by saying that this determination or this aim and direction towards the right side are not just as fast and advancing in the same way as before.

But the skeptic begins to be very touchy on this point and, if he consented in good faith that the determination towards the right did not change, he protests that he had not at all intended to consent that in changing the medium it would still make an equal progress, since the author had so often

and solemnly assured him that the determination and the moving force are altogether different and distinct; and to confirm himself in his doubt, he adds that if, in the figure on page 17, the ball was impelled from H towards B, and it were to continue its movement towards BG, then I say that this reasoning:

> "The determination of the ball along path HBG is not at all changed at point B, since it is the same, and the perpendicular movement will continue along the same line HBG; therefore this ball advances as much and just as fast below B as it did before,"

would be ridiculous, because the determination or direction of movement differs from its speed.

Why must our skeptic be obliged to concede, gratuitously and without proof, that the movement towards the right in the figure of page 18 advances equally towards the said right side, after it has changed media? It is not that this proposition could not be true, but that it *is* true only if the conclusion derived by M. Descartes were true, that is to say, if the ratio or proportion to measure refractions had been legitimately and truly assigned by him.[5] He has therefore not demonstrated it by such a doubtful and inadmissible proposition.

In a word, even if all of the objections against him may be faulty, can he put forward as true that which is neither an axiom nor deduced as a legitimate consequence of any first truth? Demonstrations which do not compel belief cannot be called such.)

And can you truly believe, Sir, if the proposition of M. Descartes *were* demonstratively proved, that its evidence and clarity would not have pierced the shadows of my understanding during the twenty years which have passed since our old quarrel, seeing that I have assured you, in the beginning of my letter, that I sincerely work to free myself from error, and that I am only too willing to surrender to an honest argument? I would indeed be delighted to establish the honor of M. Descartes at the expense of my own, and, if it were possible for me, I would like, in understanding his proof, to add before I end that[6]

[5] *Post hoc ergo propter hoc.* Can Descartes prove his assumptions by demonstrating that his conclusion succeeds?

[6] "she stood there before me with such brightness, and she gleamed with pure light across the night," Virgil, *Aeneid,* II, 589-590.

Se clara videndam
Obtulit et pura per noctem in luce refulsit.

Nevertheless, it shall be as Sir Digby and you, Sir, see fit. I submit to you both my logic and my mathematics, and I consent that you make a sacrifice of it to the memory of this illustrious man, who is no longer in a condition to defend himself; but until you have pronounced yourselves, I claim that the true ratio or proportion of refractions is still unknown and that $\theta\epsilon\tilde{\omega}\nu$ $'\epsilon\nu$ $\gamma\grave{o}\acute{\upsilon}\nu\alpha\sigma\iota$ $\kappa\epsilon\tilde{\imath}\tau\alpha\iota$,[7] in the company of so many other truths that the future will perhaps discover better than has the past.

Excuse my lengthiness, and do me the honor of believing me to be, Sir, Your very humble and very affectionate servant,

FERMAT.

[7] "It rests in the lap of the gods." Homer, *Iliad*, XVII, 514.