

Clerselier to Fermat

Wednesday, May 15, 1658

SIR,

1. I do not want to take too much time making excuses for having responded so late to your two letters, the one of the 3rd and the other of the 10th of March, since I am sure that you will easily understand that only insurmountable obstacles prevented me from responding in time to such kindness and civility on your part. Indeed, the true reason which prevented me from showing you my gratitude sooner is that I was sick in bed during almost this entire period and my mind was therefore incapable of involving itself with such elevated subjects.

But this tardiness would not matter, so long as I am able to respond to all the doubts of your skeptic today, and to fully satisfy the problems which you reference in your last letter, because, since they do not at all depend on time, the response would not have been any better or more convincing, if it had come on time.¹ Nevertheless, seeing that it is with you, sir, that I am dealing, and not with your skeptic, whose humor would be too difficult to satisfy, I am confident that I can clarify most of his doubts and show so clearly (unless I am mistaken), how he was mistaken in his judgments that by having you as the arbiter of our differences and the judge of our conclusions, I hope that you will recognize the subtleties of his and the truthfulness of mine, that is, those of Descartes.

2. First of all, I do not see how the reasoning of M. Descartes, with respect to his diagram on page 17 of the *Dioptrics* (*fig. 56*), is at all opposed to common sense, nor how the extensions that he made there from reflection to refraction is at all forced. Since the same reason which caused him to conclude

¹Yes, this is supposed to be a joke.

on page 15 that the earth CBE (*fig. 53*) could only stop the determination from high to low, and not at all from left to right, because it is entirely opposed to the first and not at all to the second; the same reason led him to conclude, in the figure on pages 17 and 18, that the determination of high to low could indeed be changed in some way by the encounter with the canvas or the water, but that which causes the ball to tend towards the right hand cannot be changed at all, because the water or canvas is in some way opposed to the one and not the other.

Fig. 53.

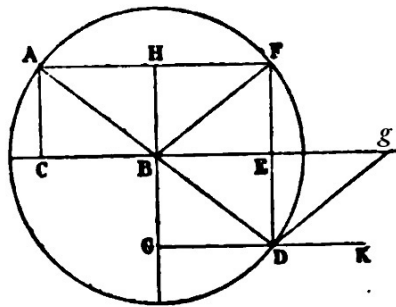
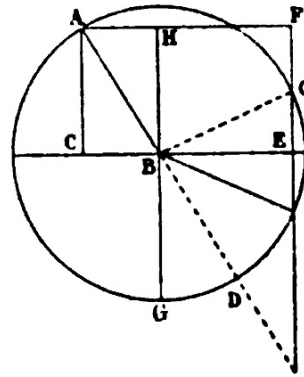


Fig. 56.



I would ask that you note the way M. Descartes speaks here, (because the solution to all of the doubts of your skeptic depends, in part, on this). He does not say simply that the determination to move from high to low can be changed by the encounter with the canvas, but only that it can be changed *in some way*.

For, indeed, it has not changed completely, since the ball does continue to descend, but is rather changed to some degree, inasmuch as the determination that a moving body has to advance towards a certain direction is changed in some way, when it is made not to move as far in that direction in a certain time as it did before: this changes the quantity of its determination.

3. What's more, the three circumstances your skeptic puts forward to avoid having to accept this result, cannot in any way invalidate it. Because, if the speed were diminished, if the medium were changed, or if the determination of high to low were not entirely prevented, but the ball were to still continue to descend, all that would not bring about a change in the determination of left to right, to which not one of these circumstances is opposed

or presents an obstacle, since this determination can remain the same even if the speed were changed, for the same determination may exist with different speeds.²

The medium also cannot bring about any change in this determination, since it can open itself to allow passage just as easily in one direction as in another, in the medium that it travels through. And although the ball continues to descend and does not come up again, as it would in reflection, this determination towards the right can just as well be made and maintained while descending as it was while ascending.

Up until now, your skeptic would be wrong, it seems, not to want to agree that the determination from left to right remains the same in refraction, after having remained in agreement without any difficulty in the case of reflection. And he has no reason to fear that we are quibbling over the explanation of this term, or that he will be obliged to affirm anything that is not proven and derived as a legitimate consequence from what came before. For M. Descartes has too carefully pointed out the difference between the determination and the movement (or, as you say, the power that moves), to be able to forget about it.

4. But here is the point which frightens your skeptic, and which makes him lose the little respect which he still seemed to attach to the name of M. Descartes. After this, he says he will hear no more banter, and that although he consented in good faith that the determination towards the right did not change, he protests that he is not required to concede that that ball, changing media, always makes an equal progress, and, as he said a little earlier, would still go just as fast towards the right after it has been assumed that the ball loses half of its speed at point B, and after M. Descartes has so solemnly assured him that the determination and the motive force are totally different and distinct.

But do you not see that what prevents your skeptic from agreeing with this, is the fact that he himself does not sufficiently distinguish between the determination and the moving force or the speed, and that he confuses them together, believing that the division or the loss that one suffers, such as to its speed, must be felt by the other, that is, by the determination towards the right, even though nothing opposes it which could have changed

²If you read the *Dioptrics*, you will see that Descartes clearly does not actually maintain a distinction between the two. Also, see the immediately preceding paragraph in this letter.

or diminished the quantity of the determination that the ball had to advance in this direction? For, if he had paid close attention to what M. Descartes said, he would not have had a problem understanding that if the speed were halved [at point B], while the determination from left to right remained the same at that point as it was before, then it is necessary (to reconcile this speed with this determination), that the ball follow line BI.

And although the headway or advancement towards the right that it makes along the path that it takes, is the same as it was earlier in the same time, and that it therefore always conserves the same determination that it had to proceed in that direction, it does not follow that it goes just as fast as it did earlier (which your skeptic seems to have always feared we would make him agree to), since M. Descartes himself admits that it requires twice the time to make as much headway as earlier. But since, along the path that it must take, it is inclined more than it was towards the right, it is not prevented from advancing as much towards that side, even though it goes two times slower.

And in my opinion, this is what makes for the entire beauty and force of M. Descartes' reasoning, to show the right path that the ball takes in this encounter, which can only be as he explained in that location, to correspond to the determination towards the right that it must maintain, and to the loss of speed which the ball underwent at B.

5. But that which has deceived your skeptic the most, is a reasoning that seems truly plausible, and is very much capable of fooling others, and even to fool oneself, if one is not careful, but which is, however, false, and contrary to the intention of M. Descartes. As M. Descartes has in the figure on page 17 (*fig.* 56), this reasoning states that if the determination towards the right hand is the the same, although the movement of the ball may be halved at point B, then in twice the time it must advance twice as much towards the right. Therefore your skeptic says that if we assume that the ball is impelled perpendicularly from H to B, and if it continues its movement along BG, and since the determination of the ball along BG has not changed at point B, and remains the same, then: when the perpendicular movement is continued along the same line HBG, it will advance twice as far in twice the time, just as fast below B as it had gone above B before. But this is absurd, since we had assumed that the ball lost half its speed at point B.

Truly, if the consequence that he infers were properly derived from what M. Descartes put forward, I would conclude, like the skeptic, that M. Descartes

would have been mistaken in his reasoning, for such an absurdity would follow from it.

But moreover, M. Descartes says something totally different than what your skeptic wants him to say: because when he said that the determination the ball had of advancing towards the right side stayed the same, and that consequently in twice the time it had to make twice as much headway towards that side, he concluded that from the fact that, although we assume that the ball loses half its speed at point B, nevertheless it loses nothing at all of the quantity of the determination which it had to advance towards the right side, to which the canvas is not at all opposed in this direction, and to which the speed which remains in the ball can and must agree with it (for otherwise the ball would bounce instead of penetrating the canvas), in such a way that without departing from the loss that it suffered, and while going slower, it still advances just as much towards the right side as it would have done if it had lost none of its speed.

But can we say the same thing of the determination of a ball which we assume to fall perpendicularly on the same canvas, namely, that the surface on which it falls, is not at all opposed to that direction, and that in losing half of its speed, it does not lose any of the quantity of the determination which it had to advance towards the direction that it was aiming at, and that the speed which remains in it can and must be accommodated with this determination, to cause it to advance in an equal time on the same route as far as it would have, had it not lost any of its speed? Certainly no one will say that this case is similar to the first, and consequently its conclusion cannot be the same.

6. Also, the entire defect in the reasoning of your skeptic comes only from the fact that he failed to consider that the same surface CBE, at which the ball loses half of its speed at point B, is also at the same time opposed to the determination of high to low, whether or not it be perpendicular. So, although the ball continues to fall and even if it were to descend along the same line when it had been impelled perpendicularly, we would not be able to say that this downward determination were the same, but rather that it is changed in some fashion, as M. Descartes says. For, the ball no longer descends with the same quantity of determination, since in an equal time it does not go as far as it was determined to go before it lost half of its speed, which is a change in the determination that it had to move in that direction.

And if you pay attention to this, all the changes of determination that

M. Descartes said will follow in the ball from the change of its speed or of the force which impels it or which stops it at B (according to the different assumptions which he has made), have all been changes in the determination from high to low, and not in that of left to right, because, as he says on line 13 of page 17, of the two parts of which one can imagine that the motion of the ball along path AB is composed, it is only that which brings the ball from high to low that can be changed in some way by the encounter with the canvas. But this is all the more reason to understand that this canvas can change the perpendicular determination to which it is entirely opposed, since it is a simple motion. We could not say that it is composed of two determinations, one of which the canvas does not oppose, as we could say of the determination of left to right, when the ball is impelled obliquely along line AB.³

Yet, what other change can occur in this determination of high to low, besides that which M. Descartes explained? He said that this ball, by continuing to descend, advances either more or less towards the bottom than it did earlier, according to the change – that is to say the increase or the decrease – that it received at B, and according to the relationship that this speed has had with the determination towards the right, which must have always remained the same, as I have said several times. That is to say that this determination would have had to cause the ball to advance as much in this direction as it had earlier.

And to conclude, as much as your skeptic wished to infer that the absurdity follows from what M. Descartes has said, it actually is found, on the contrary, that it is he [your skeptic] himself who, instead of using a good argument, has entangled himself into sophistry, by assuming that the determination of the ball in a perpendicular fall was the same, in the same sense as the determination of left to right is said to be the same when the ball falls obliquely.

7. And if, after this, you take the trouble of examining the response made by M. Descartes himself to the rest of the difficulties that your skeptic had posed to him earlier via the Reverend Father Mersenne, and to those which

³When the ball moves along AB, the determination to move from left to right is not opposed by the canvas, but when the ball moves perpendicularly to the canvas, its entire motion is opposed, and you cannot say that it is composed of two motions, where one of the motions is not opposed.

he gave satisfactorily in a letter which he addressed to M. Mydorge,⁴ which I have just recently sent you a copy of, you will find that what he says is true, namely that your skeptic is mistaken, for having spoken of the composition of movement in two different directions, and for having inferred about one that which he had proven only for the other.

I do not repeat here what he said about it, because, besides being useless, since I was there, one of my friends, M. Rohault, a wise mathematician among the most well versed that I know regarding the philosophy of M. Descartes, came to me with a response that he made to your letter to Father Mersenne, believing that M. Descartes had not responded to it (because I never showed him that letter to M. Mydorge), and thinking that you had not received any response from him, seeing that in the letter that you did me the honor of writing (which I showed him), you persist with your initial objections, and that in the letter to M. de la Chambre, in which you say that you had earlier challenged M. Descartes about his demonstration regarding refraction, you said that you challenged him *viventi atque sentienti*,⁵ but he never responded to you satisfactorily.

And since he understands all these subjects much better than I do, and since he has responded article by article to this letter of yours, I will abstain from wearying you too much with my speech, in order to leave you more time to examine the response that he has made to your letter. If he had brought it to me earlier, he would have relieved both of us: he would have relieved me from writing on a subject that exceeds my mental forces, and he would have relieved you from having to read such a bad letter. But, since it was already done, I did not want to have lost my efforts, and I thought that it would be better to tire you with this reading, and to give you by the same medium,⁶ some proofs of the care that I had taken in fulfilling my debt towards you, rather than to let you believe that I had perhaps forgotten, and that I would have been better off if someone else had discharged me of the obligation.

Moreover, Sir, I ask you to please excuse anything I may have said too freely in responding to your skeptic. I would have acted entirely differently if my writing had been addressed to you; but, far from being afraid that you would refuse to treat me justly because of this, I still take the assurance of asking you to show some forgiveness. There are some encounters where a

⁴This letter is in the Letters of Descartes.

⁵when he was alive and well

⁶The medium of letter-writing. No pun intended, here.

little favor does not at all offend against fairness, and, if you take my side on this, I can assure you that in any other encounter I will be entirely on your side, and that you can always consider that you have in me someone who is readily, Sir,

A very humble and obedient servant,
Clerselier

Paris, this 15th of May, 1658.