

# Reflections upon and a Response to the Letter of M. Fermat

Which Includes his Objections to the *Dioptrics* of M. Descartes

M. Rohault

May 15, 1658

I do not know whether Father Mersenne, to whom this letter was addressed,<sup>1</sup> had communicated it to M. Descartes, or whether, having received it, his occupations prevented him from giving a response. In either case, it appears not to have been responded to, because M. de Fermat, who had written it about twenty years ago, repeats almost the same objections in a letter that he recently wrote to one of my friends in this city.<sup>2</sup> I will try to supply some responses based on the intention of M. Descartes, and the easiest way to do this will be to follow the order in which the articles or sections appear in the letter, as I will examine it separately.

FIRST ARTICLE. — The first article contains only a compliment in which the civil mood of M. de Fermat honored M. Descartes, and to which the memory of M. Descartes is still indebted to M. de Fermat.

SECOND ARTICLE. *I divide*, etc. — Although M. Descartes fits his *medium* to his conclusion, and he divides his movement into certain determinations rather than others, it is no stranger than if a geometer decided to make use of one construction over another for the execution of a problem: and one never contests the route the geometer chooses, so long as it comes to his intended end.

Besides, M. Descartes had to divide his movement into one determination perpendicular to the surface towards which it was impelled, and another determination parallel to this surface, because, since the latter encounters no opposition, he was certain that it had to remain the same; which for him

---

<sup>1</sup>Fermat to Mersenne, December 1637

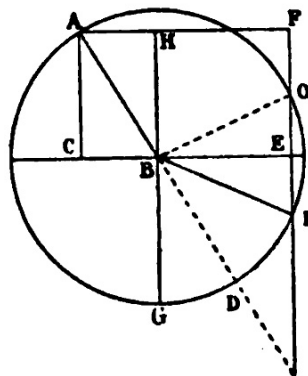
<sup>2</sup>Fermat to Clerselier, March 10, 1658

was a means of reaching a truth more easily than if he had followed another method.

THIRD ARTICLE. *I recognize, etc.* — M. de Fermat seems to be in favor of M. Descartes by admitting that he agrees with him on the question of the difference that he establishes between movement and determination, and by even trying to prove it. However it also seems that he was clever, in that he imputes to M. Descartes an opinion that he would disavow, and does this, to all appearances, in order to use it later.

It is in the second example (*fig. 56*), where he asserts that a ball, impelled from point H to point B perpendicularly to the surface CBE, does not lose its determination, because by penetrating the water or the cloth, he says, it continues to move along the same straight line.

Fig. 56.



But he must consider that the determination of a moving body must be considered as changed, not only when it leaves the line along which it was moving previously, or when it moves in the opposite direction along that same line, but also when it is moving in the same direction along the same straight line, provided that it be more or less far than it would have been determined to go in that direction. And it is in this third manner that the quantity of determination of the ball has been lessened, as well as its movement: also the surface CBE was just as opposed to the first as the connection of the parts was to the other. This is why this example, which was only made to prove a truth that the two parties did not disagree on, must be considered as null.

I would even go so far as to say that M. Fermat calls “force” or “moving force” that which M. Descartes calls “movement,” because this difference

does not appear to be of the slightest consequence in the remainder of the letter.

FOURTH ARTICLE. *I now return, etc.* — This article only contains some words of M. Descartes.

FIFTH ARTICLE. *First of all, I remark, etc.* — The lack of memory imputed to M. Descartes here, is based upon M. de Fermat's belief that the determination of high to low in the example on page 17 of the *Dioptrics* is not changed: an error which is similar to that disclaimed in the remark on the third article. And it would be of no use for proving his thought, to say that the determination along line BI is composed in part of that which causes the moving body to go from high to low, as was that which earlier made it move in the same direction along line AB. There is some ambiguity in this, and although we always consider a determination from high to low, the second is different from the first, just as ten écus are a different quantity than fifteen écus, although they are still écus.<sup>3</sup>

SIXTH ARTICLE. *But let us concede, etc.* — After M. de Fermat offered in the form of a favor something that was duly owed to him, he attempted to prove that M. Descartes did not notice that the determination from left to right was also changed; which would have really invalidated his demonstration. The reason, he says, is that one could not say that the determination from high to low was changed, unless it were because of the fact that, since the body began to move along BI, its quantity no longer had the same ratio as the one, from left to right, that it had when it was moving along AB. [[P]]

I do not know whether or not M. Fermat is being straightforward here, since he reasons like a person who, after having carried fifteen écus in one of his pockets and thirty in the other, and having lost, by I know not what accident, some of the fifteen, would realize this loss only after having seen that the remainder of the fifteen is no longer half of the sum that he has in the other pocket, after which he is given to believe, to console himself, that the contents of the latter pocket have increased, because as compensation, it now contains more than twice that which he first found lacking.

M. Descartes reasons in another manner, as would a typical young man who has not yet learned the Fifth Book of the *Elements* of Euclid. This young man would judge that he had lost some of the fifteen écus, by comparing what was left with what he previously had in the same pocket, and would not bother to make a comparison with the thirty in the other pocket. Similarly,

---

<sup>3</sup>An *écu* is a coin.

M. Descartes judges that a change has occurred in the determination from high to low, since its quantity changes: once the moving body is below the surface CBE, the determination is no longer the same as it was when it was above it. And he is right in asserting that the determination from left to right has not changed, because its quantity is the same, when the moving body is on line BI, as it was when it was carried along AB.

SEVENTH ARTICLE. *But let us again concede, etc.* — Regardless of the fact that M. de Fermat again concedes here so graciously something that he would have been wrong to contest, as is seen in the preceding remark, this article only contains some words of M. Descartes.

EIGHTH ARTICLE. *You see how he falls back, etc.* — Here, M. Descartes is accused of falling into the same fault for the second time: failing to recall that there is a difference between the determination and the movement. But this accusation is based only on M. de Fermat's taking the words of M. Descartes a little too literally: for, when he says these words: *it must make twice as much headway in the same direction*, this does not signify that the ball moves along a line twice as long as before, but rather that whatever be the line, it must be so inclined towards the right that the ball moves towards this direction two times more than it would have done. This is the correct interpretation of the words of M. Descartes, rather than the other, by which it is claimed that he is confusing two different things; and his intention was clear enough because while he says that the quantity of the determination becomes double in the same time, he assumes that the movement is only simple, that is to say that the moving body traverses a line equal to that which it would have traversed before.

That which follows in this article, and the absurdity that M. de Fermat concludes in it, is not to the disadvantage of M. Descartes, who would deny that the determination from high to low would remain the same, following what was said in the remark on the third article, and in this way all this display of reasoning goes up in smoke.

ARTICLES 9, 10, 11, 12. — I accept as true everything contained in these articles; but I fear that they have nothing to do with the subject.

THIRTEENTH ARTICLE. *Having supposed and demonstrated this, etc.* — M. de Fermat maintains that, on page 20 of the *Dioptrics*, the assumption of M. Descartes is that the increase by one-third of the movement which occurs to the ball is simply the motion from high to low or along line BG, while it has to do with measuring it along the line that it has to describe or actually travel through. And this is easy enough to understand, because, if this were

the case, M. Descartes would not have assumed as he did, that the force of the movement of the ball is increased by one-third, but he would have assumed the determination of high to low to be increased by a third, and would not have assumed anything about the movement. Therefore it must not be said that in his meaning, the ball which moves along BI moves by a motion composed of that which it had towards BD and another towards BG – from which it is desired that he assumes the quantity being one-third greater than it was along AB, but although the actual movement of the ball be one-third greater than before, leaving to reason the task of ascertaining which change must follow in the determination of high to low.

FOURTEENTH ARTICLE. *Then let us imagine, etc.* — M. de Fermat’s conclusion in this article is true according to his assumption which, as I have just remarked, is different from that of M. Descartes. So one need not be shocked if the two of them establish different proportions, whereby, consequently, one could not invalidate the other.

FIFTEENTH ARTICLE. *Moreover, the principle reasoning, etc.* — It is true that M. Descartes means that the motion of a moving body always increases by a similar quantity when it penetrates into the same medium, notwithstanding that it falls upon the surface at different angles. And this is quite reasonable, since the ease of motion depends on the nature of the body, which we assume to be such that it yields to allow passage just as easily in one direction as in another, and since the inclination of the ray of incidence depends only on the determination of the quantity of the diverse falls that can be different according to the different ratios that will hold between their determination and speed.<sup>4</sup> What M. de Fermat adds afterwards and what he claims to have demonstrated as false, is only true under what he believed to have been the assumption of M. Descartes, but which nonetheless, as I have shown, is quite different from his assumption.

SIXTEENTH ARTICLE. *It is not only that, etc.* — M. de Fermat maintains that he is not certain whether his proportion must be followed, rather than the one that he is writing against. But I have no objection to accepting his own, if the acceleration or deceleration of the movement depended only

---

<sup>4</sup>The lack of clarity in this sentence is translated from the original French: *Et cela est bien raisonnable, puisque la facilité de se mouvoir dépend de la nature du corps que l’on suppose tel qu’il se peut ouvrir pour faire passage aussi facilement vers un côté que vers un autre, et que de l’inclinaison du rayon d’incidence dépend seulement la détermination à la quantité de laquelle les diverses chutes pourront apporter de la variété selon le rapport qu’auront entre elles la détermination et la vitesse.*

on the surface common to the two media through which the moving body travels: but since this surface would only be able to deflect the movement and since it is the second body which speeds it up or impedes its motion, the proportion of M. Descartes must be retained.

When it pleases M. de Fermat, we should like to know his thoughts on refraction; but I can already say in advance that the summary you let me see of his letter to M. de la Chambre seemed very ingenious and worthy of him.<sup>5</sup>

If you show him this letter, I ask you not to mention my name, or, if you decide that it is appropriate to tell him, then I also ask you to let him know that it is not only recently that the fame of his name has reached me, that I very much honor his merit, and that I would be honored if he would be so kind as to consider me among the ranks of his very humble servants.

---

<sup>5</sup>Fermat to M. de la Chambre, August, 1657.