Fermat to Clerselier

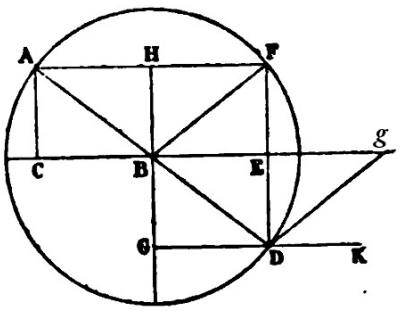
Sunday, June 16, 1658

Sir,

1. We last left the ball of M. Descartes in quite a predicament.[1] It is in the figure on page 19 of the *Dioptrics* (*fig.* 56), where it was using all of its effort to leave point B for the honor of M. Descartes. But by following the reasoning of the author, it found all the exits closed, and indeed we cannot now give it any assistance, unless we change the orientation of his logic.

Let us again take up the figure on page 15 (*fig.* 53) and let us assume that the ball which goes along the line AB loses half of its speed when it arrives at point B.

Fig. 53.



If it continued in the same medium, and if the plane CBE were not at all opposed to it, it would still move along the straight line towards D, with, however, this difference: that it would take twice as long to go from B to D as it took to go from A to B. But if, still assuming the same diminution of

[1] See the letter of June 2nd.

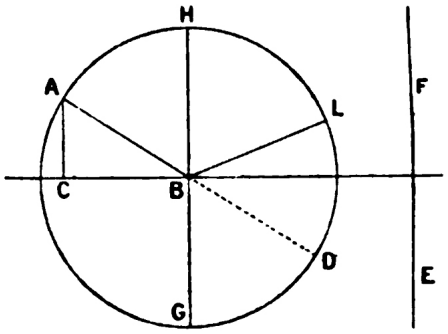
speed at point B, we now assume that plane CBE (impenetrable to the ball) is found between the two points A and D, and prevents the ball from passing below it, I say that it will reflect at equal angles just the same as if its speed and motion had remained the same.

For, since the interposition of the plane prevents only one of the parts of which the determination is composed, and that of left to right stays the same, the ball will therefore advance as much towards the right as it would have done below, if the plane had not prevented that route. Yet, if the plane CBE presented no obstacle, the ball, whose speed is halved at point B, would expend twice the time from B to D as it had from A to B, and upon arriving at point D, it would have advanced towards the right to E; it would therefore expend double the time advancing from B towards E as it took to advance from C to B. And there is the same ratio between AB and BC as between BD and BE, because the angles ABC, DBE, on the two lines AD and CE, are equal, and consequently the triangles ABC, DBE are similar.

We can make the same reasoning above, if from point E we erect perpendicular EF, and say that, when the ball will be at one of the points of the circumference, such as F, it will have spent twice as much time as it had spent from A to B, since the plane which we now suppose to be between the two still does nothing new in preventing the determination from above to below. Therefore, the determination of left to right will then be marked by the same point E, and consequently, as FB is to EB, so is line AB to BC. Whence it follows that the angles ABC, FBE will always be equal, no matter in what manner or in what proportion the speed or motion change.

2. If M. Descartes had made sure that however the speed changes at point B, reflection is still made at equal angles, he and his friends would not have been in difficulty in drawing the ball from point B, where they saw it unhappily engaged in the example of my previous letter. He would not have maintained that, although the speed changes at point B, the ball would nevertheless goes on to advance towards the right as much as it did before. He would not have deduced the proportion of refractions from a foundation that was not only uncertain, but also false, and finally, he would not have failed, in the figure on page 19 (*fig.* 89), to determine at which angle the ball was to be reflected at point B towards point L.

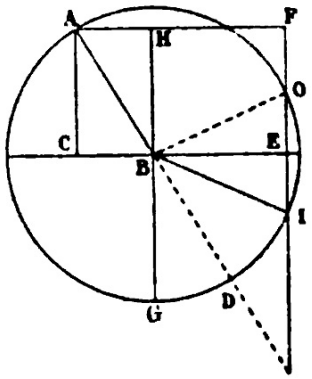
Fig. 89.



For, although it seems, by his discussion and indeed the inspection of the figure, that he understood that this reflection is made at equal angles, he left a small doubt in the minds of his readers, who can reasonably question whether or not, in the example of M. Descartes, the ball decreases its speed at point B. If it does decrease, the reflection cannot be made at equal angles, following the reasoning of M. Descartes. For if the ball does not decrease its speed at point B, is there anything more contrary to the inviolable laws of pure Geometry, which does not allow going from one extreme to the other without passing through all the degrees in the middle?

3. Yet, M. Descartes and his friends maintain that the ball, which is impelled onto the water or onto the cloth, loses its speed equally at whatever angle through which it may move through the surface, and that this diminution begins at point B. How is it possible to conceive that, from the very first angle at which a ray may be reflected, its speed never decreases at all, and that it were not possible to find any greater angle from which it could have diminished by some quantity that would always remain the same? Would it not be more geometrical and more natural to maintain, following the thoughts of M. Descartes, that the diminution of the speed is made unequally, that this diminution is the greatest in the perpendicular fall from H towards B and that it continues to lessen to the degree that the inclinations vary to the point of becoming null? Perhaps M. Descartes believed this occurred in reflection. But, because we have just proved that, whether the speed increases or decreases at point B, reflection does not cease to be made at equal angles, we do not need to trouble ourselves with scrutinizing more closely the secret that nature uses in reducing the speed of the ball, either equally or unequally, as the inclinations change.

Fig. 56.



4. But what will become of the reasoning that must be made below plane CBE, on page 17 (*fig.* 56), for example? It will be the same as the preceding: for, whether the speed diminishes at point B either by the encounter with the canvas or by some other means which might come from somewhere else, it is all the same thing. And since in the figure on page 17 the ball penetrates the canvas and at point B the speed is halved, it can never have the same determination towards the right that it would have had if there was no canvas, though the speed had been halved at point B, and if it had continued always along its route towards the straight line ABD.

You will reply: But, in that case, the determination from above to below would also not change by contact with the canvas. I confess it, and in order to remove and plainly clarify this difficulty, it is only necessary to say that you will never derive anything else from the reasoning of the motions and of the composite determinations of M. Descartes, except that reflection is always made at equal angles and that the penetration of the second medium must always be made in a straight line. This refers also to what you were saying yourself in your last writing, that the ball has always the same ease of penetrating the second medium in all sorts of inclinations;[2] from which it must follow, by application of the reasoning of M. Descartes, that in all types of cases reflection will be made at equal angles, and that the penetration will in all cases be made along a straight line following the same laws and corresponding exactly to the movement above with equal angles.

"But will there then be no refraction?" you will say. I reply that the movement of the ball and refraction only resemble each other in the imaginary

[2]Independent of the angle at which the light hits it, that is.

comparison of M. Descartes, and that in last resort, if the detour of the ball by passing through the second medium is true, it is necessary to find the reason elsewhere than in the composition of motions, which in this encounter will only ever produce a dialectic circle.[3]

Whichever side you would take, it will be necessary to examine the secret principles which nature uses to produce refraction, and if the one that I have touched upon in my letter to M. de la Chambre does not satisfy you,[4] I encourage you to discover better ones yourself, that this old dispute may finally be put to rest with the full and complete discovery of the truth.

I am with all my heart, Sir,

Your very humble and obeisant servant,

Fermat.

[3]So, refraction does occur, but it has nothing to do with the motion of a ball through a poorly woven canvas!

[4]Least time. See Fermat's August, 1657 letter to de la Chambre.