Clerselier to Fermat

Wednesday, August 21, 1658

Sir,

1. I find it more difficult to write to you than the last time: also, have you changed your position from being a judge to a disputant? When I have only to defend before you the cause of M. Descartes against your skeptic, I did not promise myself a worse success than that which I have had: I had a good cause to defend, subtleties to clarify, and a clear-headed judge to listen to me and to render judgment. But, when I consider you come down from your seat to present yourself in person against him that I defend, the respect that I owe you in whatever state you appear, the great esteem that I have always had for you, an esteem which increases more and more as I become more acquainted with you, and the slight experience that I have in the matter of our discussion in comparison with that which you have acquired, all this astonishes me and makes me not know how to reach a *denouement* of this quarrel.

I would tell you, however, that if I wanted to act with less frankness than is demanded by the honest manner with which you deal with me, I could make use of an exception which perhaps appears legitimate and acceptable, by accepting everything you say as true, and by pretending that all of this does not attack M. Descartes and does not oppose in any way his doctrine of reflection and refractions.

For I wish that the ball in the figure on page 19 of the *Dioptrique*, according to the assumption that you make in your first letter,[1] finds itself prevented (as you say, no doubt, in a playful way) from finding any exit to make its way out; and I indeed wish that the passport that you have given it in advance in your second [letter], out of fear that we had not enough credit

[1] June 2, 1658.

to furnish it with one ourselves, and even if the route that you have done the honor of marking for it in this place,[2] is so easy and convenient that it has no difficulty following it, what could one conclude from this against M. Descartes? He only brought in the examples of the ball to explain certain particular effects of light, namely that of reflection which is always made at equal angles, and that of refraction which always occurs in the same way in the same media and which changes according to the proportion between the medium which it leaves and the medium which it enters, which means that sometimes it approaches and sometimes recedes from the perpendicular: which, I say, provide no occasion to explain the case you propose, because it has nothing to do with his intention.

2. There are only three [cases] that can apply here, and he [Descartes] has explained all three, in such a clear and simple manner (in my opinion), that it is only those who insist on going beyond him that have trouble with it.

The first case, which explains reflection, is that of a ball which, being impelled along line AB, obliquely encounters an impenetrable and unmovable hard body in its path. What could be more simple and clear than that this ball, which loses none of its speed, must bounce at equal angles, i.e. it re-ascends just as quickly as it descended and advances as much as it previously did towards the side to which the hard body is not at all opposed?

The second, which is related to refraction when it moves away from the perpendicular,[3] is that of the same ball which, being impelled as above, also obliquely encounters across its path another medium, into which it penetrates and which causes it to lose a portion of its speed. What could be more clear or more simple than to say that this ball, not being able to move as quickly as it had before, must therefore conserve the determination that it had earlier of advancing in a certain direction, towards which this medium is not at all opposed – which conservation is not resisted by the loss of speed that it underwent, and can indeed be accommodated to it? Why should we want to oblige this ball to do more than it must, when nature does nothing in vain?

Finally the third case, which relates to refraction when it approaches the perpendicular, and the only one which remains for M. Descartes to clarify, is happily explained by the same ball which, being impelled as before, also

[2] June 16, 1658.

[3] E.g. light moving from water into air.

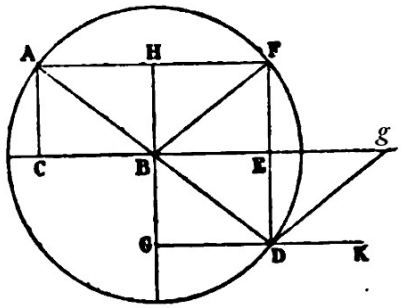
obliquely encounters across its path another medium, into which it penetrates with an equal ease in any direction and which increases its speed by a certain quantity. What can be more simple or more natural than to say that this ball, having to move more quickly than before along one of its directions, does not therefore advance further along that direction to which this body, by which its speed was augmented, is not at all opposed?

3. The case that you propose in addition to that in your first letter is superfluous and cannot explain anything about these phenomena of light. And consequently, it is of no consideration here and whatever disadvantage may follow from it, it cannot prejudice that which M. Descartes had proven earlier, and by which he has so intelligibly explained these marvelous effects of light which do not fail to be true and in accordance with his demonstrations, when your supposition will be difficult to explain by these principles, which I do not despair of doing, but I dread having to explain it according to yours.

But, since this is the nub of our question, I must once and for all clarify a point which to you seems not to have been proven by M. Descartes, because his proof is not purely geometrical but is rather based in part on some principles of nature that are so clear that they demand no explanation.

4. These principles are: 1st, that each thing remains in the state that it is in so long as nothing changes it; 2nd, that when two bodies which have incompatible modalities encounter each other, there must truly be some change in these modalities to make them compatible, but this change is always the least possible; 3rd, that a body cannot resist or cause a change in another except insofar as it is opposed to it.

Fig. 53.



Then, if a ball moves from A towards B in the figure on page 15 (*fig.*

53), with a given speed, it will always continue to go with the same speed towards that side if nothing changes it. But if you oppose it with the hard, impenetrable, and unmovable body CBE, then for the modalities of these two bodies, the one which directs the ball towards D and the other which opposes this route, but does not oppose its speed, are incompatible, it results that this change must come from a change in one of these modalities, but the least which could be. That is why the ball will change its determination and maintain its speed, and inasmuch as body CBE is only opposed to one of the two determinations of which the ball is composed with respect to body CBE upon which it falls, namely to that which was making it move downwards and not that from left to right; this body can only bring about a change in this and not the other, to which it is not at all opposed. This is why he [M. Descartes] requires that the ball go back up and be allowed to continue to advance towards the right as it did previously: a situation in which nothing changes, the modality of its body having nothing incompatible and opposed to it.

All that remains to be added to this reasoning is that which pertains to Geometry, and the proof will be made. If you do not call this proof demonstrative, then I do not know what reasonings would have to be used to compose such a proof; but, for myself, I am satisfied with demonstrations such as this.

Yet, the same reasoning that I have just made can be accommodated to the figure on page 17 and to that on page 19 and to all cases which can be proposed, and I see nothing different besides the different assumptions: namely whether body CBE is hard or liquid; penetrable or impenetrable; the speed decreasing, increasing, or staying the same; or the ball continuing to descend or being required to re-ascend; and even whether or not a body be opposed to the path of the ball.

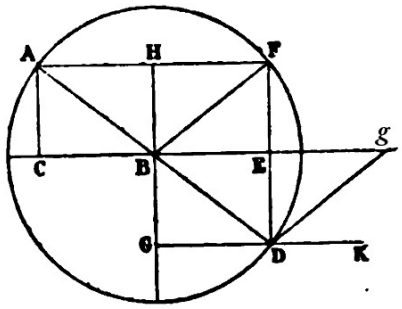
5. Let us now examine these cases one after another following these principles, and let us see what conclusions we reach; and I am sure that we will not find that things must go as you say, but rather as M. Descartes says, and this will at the same time answer all your new difficulties.

First, you quite properly say, at the beginning of your second letter,[4] that if we suppose that the ball which goes along the straight line AB decreases its speed by half when it arrives at point B, it will still go along the straight

[4]June 16, 1658

line towards D, if it continues to move in the same medium and if the plane CBE is not opposed to it: with only this difference, that it will take twice as much time from B to D as it earlier took from A to B, and that is because a body must always remain in the same state if nothing changes it. Yet, there being nothing that changes about the ball except its speed, nor anything by which the determination would be altered more towards one direction than the other, it results from all this that it must continue along the same line, only going more slowly along this determination: just as when a body falls perpendicularly from air into water, it continues to go according to the perpendicular line, only slower, accordingly as its speed is decreased by the encounter with the water.

Fig. 53.



If therefore I had been in the mood to quibble (which will never happen so long as I am dealing with a person of honor and merit such as yourself), I would have been able to deny that the case that you propose was conceivable and admissible: namely that a moving body, without changing media, can all of a sudden change from one speed to another without passing by degrees between them. This you yourself hold contrary to the inviolable laws of pure Geometry and is itself contrary to the law of nature that every body always continues to remain in the same state as long as it can, and that it never changes except by encountering other bodies. [How on earth could it be] that a body could, after arriving at point B, all of a sudden lose half of its speed, when it encounters nothing which could cause this loss? But I would like to allow you all your assumptions and deny you nothing, except that which could not possibly be admitted without overturning all of nature's laws and all the clear and simple notions that are within us.

6. Let us move on to your second assumption, which to my thinking is

one of the most clever of this type that could be made. Without a doubt I would have had difficulty in perceiving the subtle argument: since I am accustomed only to follow the very simple paths of my reasonings, I distrust anything I see that strays from them.

After this, you assume that, the ball losing half of its speed at point B as before, the impenetrable plane CBE finds itself in between and prevents the ball's passing below; and you say that the ball will reflect at equal angles just as it would have if the speed had remained the same. And certainly I confess that you prove it in the most ingenious manner possible; but also permit me to tell you that it is specious and allow me to show you in what way I think you are mistaken.

As for the above example, I maintained the agreement that the ball, losing half of its speed at point B, with the sole difference that it goes slower by half, that was because, with no change of medium and no plane being opposed, one could not say that the determination of the ball following line AB was composed of two determinations, nor when a ball falls perpendicularly on a plane. But here, where you suppose that the plane CBE is opposed to it, it is certain that from its standpoint the determination of the ball along route AB is composed of two determinations, one which makes it descend towards the plane, and the other which makes it advance towards the right or horizontally, and that the plane is opposed to the former and not to the latter.

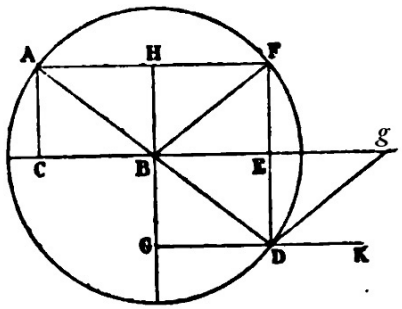
7. Now, one of the following must be true: either you assume that after the ball has arrived with two degrees of speed, for example, from A to B, that at point B it encounters the plane CBE which makes it lose half of its speed; or, you rather assume that, without the plane contributing to it, after having lost half of its speed at B, the ball encounters the plane CBE. And if I have properly understood the meaning of your second letter, it is principally inclined to the latter; but remark here again in passing that I am conceding to you more than I must: for how can one conceive how a ball may lose half of its speed at point B without encountering any body which could cause this loss!

8. In the first case, it is easy to see that it is only necessary, as you have done in your first letter,[5] to transfer the reasoning of the figure on page 17 above the plane, and say that, since the ball loses nothing at all of the

[5]June 2, 1658

determination that it had to advance towards the right, it must (all other conditions remaining constant) arrive at point O, as you have quite properly shown. That is why I would not dare to say, as you do: "Why pray tell will the reasoning of M. Descartes be conclusive below, but not above? Does that which is a demonstration in one case, become a paralogism in the other?" Doubtless, the answer is no: both can equally well be concluded.

Fig. 53.



9. In the second case, the ball can follow the path that you have marked in your second letter,[6] and always reflect at equal angles, no matter in what manner and in what proportion the speed or the movement change at point B: but not truly for the reason that you give. For the same proportion need not be maintained by a ball which, obliquely encountering across its path an impenetrable plane, is forced to reflect, as that which is maintained by another ball which we do not suppose encounters it, and which must follow the same laws as that which encounters it perpendicularly, because a ball which does not encounter a plane has only a single determination: it goes neither to the left nor to the right, although a ball which falls obliquely upon a plane always has two determinations, one to which the plane is opposed, and the other not: and this circumstance must change the result, according to the previously stated principles.[7]

But here is how the ball can follow the path that you have indicated, and reflect at equal angles, to wit: it must be assumed that the ball, having lost half its speed (or any other portion of its speed that you would like) at point B, then begins at point B to follow the route that it would have followed if it

[6]June 16, 1658

[7]That is, the decomposition into two determinations can only be made when intersection with a surface provides the occasion and the direction, to make such a decomposition.

had begun to move at that point with the remaining speed.[8] Yet it is established that if, without regard to the line segment AB that it traversed with two degrees of speed, it began to move at B with the speed that we assume it still has, and in the same direction that it truly had at point B, it would go towards D with one degree of speed and would arrive there in twice the time that it took it to move from A to B, if nothing opposed its movement. And if, rather than being opposed by the impenetrable and unmovable plane CBE at point B, it were instead opposed at point D, it is clear from what we have said above, that the plane will prevent it only from passing through it and not at all from advancing towards the right, and will neither decrease nor increase the speed with which it would have arrived coming from point B, it would bounce it towards g and would make an angle of reflection gDK equal to that of incidence BDG, which would be found equal to that of the first incidence ABC. Yet at point B the same change in the determination of the ball must occur as happens at point D if the plane CBE is opposed at this point, since from point B the ball has the same speed and the same determination as it would have at point D after traversing line BD.

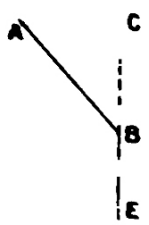
10. And therefore, the ball, following your assumption, must rebound at point B at an angle equal to that of incidence: not, as I have said, for the reason that you have given, because it is not true that while the interposition of plane CBE only prevents one of the parts of which the determination is composed, that from left to right remains the same as it was when the ball had no plane that opposed it; because, in this last case, the ball only had one determination and it cannot be said that it was proceeding towards the right. That is why the conclusion that you draw from it is not true either.

Therefore, you say, the ball must have proceeded towards the right just as much above as it would have below if the plane had not prevented its path; and since, when it would be at point D below, it would have proceeded in two moments towards the right from B to E, similarly, to advance in two moments just as far above towards the right, it must go towards point F which is as advanced towards the right as is point D, and which cuts the circle above in the same proportion that D cuts it below, and makes an angle of reflection equal to that of incidence. Therefore this whole proportion from left to right that you say must be maintained above as it would have been below,

[8]That is, if it had started its motion at B, but with the remaining speed, rather than the original speed at A.

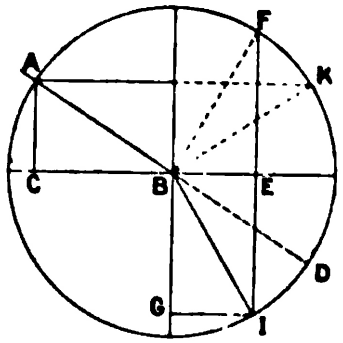
if the plane CBE had not impeded its path, is nothing but an imaginary proportion, since below, when there is no interposing plane, the ball has no direction towards the right, this direction or determination towards the right being always relative to the interposed plane. And for example, if plane CBE had been opposed to it in another direction as in the following figure (*fig.* 92), where would your entire reasoning be with respect to the right? So this must be true of your assumption itself and any other, for the reason that I said, which is in conformity with the laws of nature and to the above-established principles.

Fig. 92.



11. In order to further clarify this point, let us assume for the third case, as M. Descartes has done at the end of page 19 of his *Dioptrics* (*fig.* 93), that the ball, having first been impelled from A to B, intersects plane CBE at point B, which increases the force of its motion or its speed by a third [sic], so that it could afterwards make as much headway in two moments as it made in three before. And it clearly follows that it must rebound at F, because the determination towards the right could not be increased by the plane CBE to which it is not at all opposed: and not at K as it would do if your reasoning were true, but which cannot be, because it is contrary to the laws of nature and contrary to experimentation, which shows that the reflection of a ball and of other similar bodies, which are not perfectly hard, or which fall on others which slow down their motion, never occurs at equal angles. Therefore the softest balls do not bounce as high and do not make [reflective] angles as great as do those that are harder.

Fig. 93.



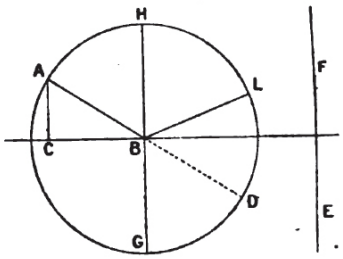
And note that since it is naturally easy to conceive that, in order to produce a reflection at equal angles, the motion must in no manner be increased or decreased by the intersection of the plane, it seems that reason must also naturally orient us to believe that, when this plane increases or decreases it, the angle of reflection must be of a greater or smaller proportion with respect to the angle of incidence, and not that it must always be equal, as follows from your reasoning, which you must begin to suspect, even though it is very ingenious.

12. But, tell me, what will happen then to the ball in the supposition that I made at the end of my first letter,[9] on the figure on page 19? For here is the point of difficulty, and it must finally be removed from this fatal point where it appears to be in a very uneasy position. That is also what I mean to do now, to the honor of M. Descartes and without changing the bent of his logic, by using, in the case that you propose here, the same reasoning that I have already used when I passed to your second assumption.

If then the ball, having arrived at point B, obliquely encounters the hard, impenetrable and unmovable plane CBE, and if at point B it loses such an amount of its speed that the line FE, drawn as in the preceding examples, lies outside the circle AD, I say that: either you mean that the plane CBE contributes to the loss of speed, or you mean that it contributes nothing to it.

[9]June 2, 1658, **4**.

Fig. 89.



If it contributes nothing to the loss of speed, then we can only imagine that the ball, after having lost two-thirds, for example, of its speed, and since in this condition it has a direction determined to go towards D in a certain time, in proportion to the force or speed which it still has, and consequently to also proceed following this force of a certain quantity towards the right, with respect to the plane CBE that is opposed to it (vertically), but which is still not opposed to this direction toward the right, then it must bounce at point B as it would at point D, just as I have said above. And there you have the route that I would have laid out for it, which is in conformity with yours, but by another reason which does not require me to change my logic.

But note that this assumption is itself impossible: that a ball could lose two-thirds of its speed without encountering any other body which could make it lose it.

Now if the body CBE does contribute to the loss of speed, this could not be if we assume the body CBE to be perfectly hard, impenetrable, and unmovable. For the movement of the ball can only be diminished by the encounter with a body, to the extent that the ball transfers to it some of its movement; and if it transfers it, this can only be in the direction to which the body CBE opposes it and consequently it can only transfer its movement to it along this part of its direction which makes it move towards it; and the encounter with the body CBE (which we must assume to be completely uniform) can never decrease its direction towards the right or parallel [to the body]. Now it is easy to conclude that, if the ball at point B has transfered to body CBE all the movement, which makes it bend downwards, it must continue its parallel movement and roll along it, proceeding just as much towards the right as it had earlier.

13. If, notwithstanding this, you wish to make this impossible assump-

tion, against all reason, that it loses such a portion of its speed at point B such that it cannot proceed towards the right as much as it did before, and consequently that it has also lost a portion of its motion which causes it to proceed towards the right, then I will tell you that it will roll on its diameter with the speed which it still has, just the same as, when you assume that without encountering any plane whatsoever it still manages to lose some of its speed, it must continue its path along the same straight line that it had begun to traverse. And thus the same thing will happen to this ball as if, having been moved with a given speed along the plane CBE, it happened that at point B (by an impossible assumption and without any cause whatsoever), it would lose a portion of its speed: it would continue along its path on the same plane with the speed which it still had.

But note that, to find something defective in the reasonings of M. Descartes, it is necessary to use impossible assumptions, and consequently it will not be a marvel when from an assumed impossibility an absurdity follows.

14. By all of the above, it seems that everything that you say in your second letter fails on its own account and does not require a response: namely that "if M. Descartes had made sure that in whatever manner the speed changes," that is to say that it increases or decreases, "at point B, reflection is not prevented from making equal angles, he and his friends would not have had difficulty in drawing the ball from point B, where they saw it unhappily engaged in the example of my previous letter. He would not have maintained that, since the speed changes at point B, the ball nevertheless goes on to advance towards the right as much as it did before. He would not have deduced the proportion of refractions from a foundation that was not only uncertain, but also false."[10]

All this, I say, being no longer motivated by valid reasonings, destroys itself, just like what you add at the end of the same letter: *viz.*, that the second medium being able, as I have said, to open with equal ease in all directions to allow passage for the ball, and that the ball always having the same ease of penetrating the second medium at any angle, it must follow, you say, "by application of the reasoning of M. Descartes, that in all types of cases reflection will be made at equal angles, and that the penetration will in all cases be made along a straight line following the same laws and

[10]June 16, 1658, **2**.

corresponding exactly to the movement above with equal angles."[11]

15. Now, if I have made myself clearly understood, you must draw other conclusions from the principles of M. Descartes [than those that you came up with] and, if I am not mistaken, you must have recognized the error of the reasoning with which you arrived at them. Therefore never again say that the motion of the ball and refraction are only similar in the imaginary comparison of M. Descartes; for it is perhaps the most true and the clearest that could be brought to bear to explain it. But, for this, the ball must be considered without weight, without size, without shape and without any change in speed as it moves along all the lines that it traverses:[12] all of which could cause an infinite variety in reflection and refraction of a ball, but, inasmuch as they have nothing to do with the action of light, M. Descartes did not consider [these properties] in the motion of the ball of which he speaks.

Mainly, he did not consider this circumstance that I urge you to consider, which is the most common and which can give the greatest occasion to doubt what M. Descartes said, that is, to know that as far as the medium that the ball traverses ordinarily removes a part of its speed at each moment that the ball moves through it, from that it follows that a ball can have lost at the point of reflection half (or more or less) of the speed that it had at the beginning, and that it would not stop it from reflecting at equal angles, because at the moment that it comes to touch the plane, its speed has already been diminished by the medium that it passed through and the direction that it then had does not prevent its determination to go along the same line where its first direction carried it when it left the hand or the racket, provided that its weight, size, and shape have not changed the while.

16. What I say about speed when the medium decreases, must also be understood to hold when it is increased at every moment by its weight: as, when a ball falls along an inclined plane, it will also be bounced off at equal angles, even though its speed is increased at the point of reflection; and that for the same reason, namely that this increase does not come from the plane, but is received before encountering it.

And thus you see how the principles of M. Descartes are solid and his reasonings well ordered; which demonstrates that the true reason of refrac-

[11] June 16, 1658, **4**.

[12] It would seem that M. Descartes' balls exist only in his imagination!

tions must be derived from composite movements and determinations, by examining them as M. Descartes did. And, truth be told, M. Descartes was a man of too much good sense and who kept too close an eye on things, to fall into these obvious or gross faults. And it seems to me that he gave us reason to hold a high enough opinion of him to believe that *we* are mistaken in not understanding his meaning and his reasoning, rather than to believe that *he* had erred, especially when the error into which we believe he has fallen is so gross and apparent.

17. I will only add that, since the different experiments that M. Petit has performed here (that you know of) with all sorts of transparent bodies, are all in agreement with the proportion that M. Descartes has found, it is probable that the reasons which have led him to discover the proportion were truthful. To think that the same reasoning could lead so precisely to the truth in so many different cases, and yet be false!

If, after all this, you still wish not to admit the conclusions that I have drawn from the principles that M. Descartes has established, at least receive the conclusions of this letter as true and believe that, if my reasonings are faulty, the proclamations of my heart are sincere when I assure you that I wish to be etc.