

# Clerselier to Fermat

Saturday, May 6, 1662

SIR,

Do not think that I am writing to you today with the intention of disturbing the peace that you present to all Cartesians. The conditions under which you have offered peace to them are too advantageous, and for me in particular too honorable, not to accept them; and if everyone who had ever had a quarrel with his teacher were as sincere as you, you would soon see peace established everywhere to the satisfaction of all parties.

There still were two types of mind to satisfy on the subject of refraction:

You took the first minds, little versed in Mathematics, who were unable to understand a reason taken from the nature of composite motions, and you have brought them to reason, by proposing to them another principle, more plausible in appearance and more accessible to them, namely, that nature always acts along the shortest and simplest paths.

From the other minds, who were too mathematically knowledgeable and who could not accept the pure and simple reasons of metaphysics, which must however necessarily be joined with these principles to give them the force of conviction, you have removed this obstacle by deriving your principle by a purely geometric reasoning.

And since these two types of people were without doubt more numerous than other types, you also easily deserve a greater part of the glory for such a beautiful and important discovery.

I do not envy you this discovery, and I promise you sir, to publish every where and confess openly that I have seen nothing as ingenious nor as well-conceived as the demonstration that you have brought forward. Please allow me only to put forth here the reasons that a somewhat zealous Cartesian might allege in order to maintain the honor and right of his master, and to

not relinquish so hastily to another his position, or to give him the initiative.

1. The principle that serves as foundation for your demonstration, namely that nature always acts by way of the shortest and simplest paths, is but a moral principle, and not at all physical, which *is* not and *could* not be the cause of any effect of nature.

It *is* not, because it is not this principle that makes nature act, but rather the secret force and the virtue that is in each thing, which is never determined to a particular effect by this principle, but instead by the force which is in all causes and which comes together in a single action, and by the disposition which is actually found in all the bodies upon which this force acts.

And it *could* not be such [the cause of any effect of nature], because otherwise we would assume nature to have knowledge: and by “nature,” here we mean only that order and that law established in the world as it is, which acts without foreknowledge, without choice, and by a necessary determination.<sup>1</sup>

2. This same principle must put nature in an unresolved state, not knowing what to do when she must pass a ray of light from a rare body into a denser one. Because, I ask you, if it is true that nature must always act by the shortest and simplest pathways, and since the straight line is undoubtedly both the shortest and the simplest of all, then when a ray of light has to travel from a point in a rare medium to a point in a dense medium, is it not the case that nature must hesitate? For if you wish her to act by the principle of following a straight line immediately after the break, then isn't your path the shortest in time, while the straight line is shorter and simpler in measure? Who will decide, then, and who will pronounce himself on this matter?

3. Since time is not what moves things, it also cannot be that which determines movement, and once a body is set into motion and is determined to go in some direction, there is no apparent reason to believe that a shorter or longer time would force this body to change its determination, since time does not act on it and has no power over it. But, since the speed and determination of the movement of this body depend entirely on its force and the disposition of its force, it is much more natural, and in my opinion, much

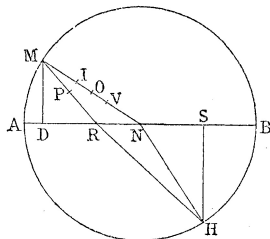
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<sup>1</sup>It appears that “laws” must act moment-to-moment, determining the future only from the immediate present.

better physics to say, as M. Descartes does, that the speed and determination of this body change because of changes which occur in the force and in the disposition of this force, which are the real causes of its movement, rather than to say, as you do, that they are changed by a design that nature has to always proceed by the pathway that it can travel through the quickest, a design which she cannot have, because she acts without knowledge, and which cannot have any effect on the body.

4. Since the straight line is the only one that is determined, nature will orient all of her movements in straight lines, and even though sometimes a moving body actually describes another path, nevertheless, if we consider in turn all the points that it has traversed, they are much more the points of all the straight lines that it has left successively than those of a curve that it tends to describe, and it has traversed those points rather than the others, since as soon as the body is left and abandoned by the force which moves it at each point, it tends to follow the straight line to which this point belongs, and not the curve that it had been describing (*fig. 101*).

Fig. 101.



This being the case, if it is a matter of carrying a ray of light from point M to point H, it is certain that nature will send it straight along line MH if possible, and in fact, when the medium is similar and equal, she never fails to do so. But when the medium the light travels through changes its nature and offers more or less resistance to its passage, what shall be said to change its direction when it enters the new medium? What shall we suspect is the cause of this?

Could it be brevity of time? Certainly not. For when the ray MN arrives at N, it must make no difference to it (if it acts according to this principle) which point on the circumference BHA it goes to, since it would take just as long to get to one as it would take to get to any other. And since this reason of brevity of time cannot carry this ray more towards one location

than another, it would be reasonable to say that it would instead follow a straight line. After all, in order to choose point H in preference to any other point, it would be necessary to assume that this ray MN (which nature was only able to bring to point N by the tendency for a straight line), would then have to remember that it came from point M with orders to go seek out, once it reaches this new medium, the path that it could have traversed in the least time to arrive at H: this truly is pure fantasy and is in no way founded on Physics.

What then will make ray MN change direction (once it has arrived at point N) at the encounter with another medium? What else could it be, other than that [cause] put forward by M. Descartes? He says that the force that acts on and moves ray MN changes its disposition when it encounters a different disposition to receive its action in this medium, and it brings the direction of the ray into conformity with this new disposition that it has.<sup>2</sup> And, with respect to the point of the encounter with the new medium, only the force carrying the ray downwards experiences a change in the way the medium receives its action, which is a difference between the medium that it is leaving and that which it is entering (the force that moves it towards the right does not experience any change, because this medium is not opposed to the ray in this direction),<sup>3</sup> the change which occurs to the manner in which the action of the force which carries it downwards is received at the point of the encounter, also changes the direction of the ray and deflects it towards the direction it is attracted, according to the proportion then found between the action of this force and the action of the other.<sup>4</sup> And this seems so clear to me, that no more difficulties can remain.<sup>5</sup>

5. Even though it ostensibly seems more reasonable to believe that light finds passage through rare bodies more easily than through dense ones, as you assume, based on the experience of all the physical bodies which undeniably do move more easily through these sorts of media, nonetheless, it seems to me that it is still more reasonable to believe that the bodies which enter

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<sup>2</sup>Excuse the repetition of “disposition,” in which we have followed Clerselier. He is saying that the force is more or less easily able to put itself into action in the new medium, and that it will change the direction of the ray accordingly.

<sup>3</sup>So, somehow the medium’s ability to change light’s ability to move is entirely at the surface?

<sup>4</sup>The proportion between the forces downwards and to the right.

<sup>5</sup>The reader who does not find this clear is not alone.

into media which pose more resistance to their passage than the media that they left, as you assume dense bodies do to light, would strive to avoid the medium and would enter into it as little as possible.

This is what experiment confirms: when a ball is impelled obliquely from air into water, far from continuing its movement along a straight line, and farther still from diving in still more to approach the perpendicular, it actually moves away from it as much as it can, approaching the surface [instead of the perpendicular]. Surely you have recognized the force of this objection, although you call it weak, but you would be incapable of resolving it without the principle of M. Descartes, which completely destroys your own [principle].

For if, with your very principle, the ball must move away from the perpendicular, why would light move towards it? And if the ball does not follow your principle, as indeed it does not, why would light follow it? Does this not rather show, by these examples, that nature does not follow your principle?

6. This pathway, that you claim to be the shortest because it is the quickest, is only a pathway to error and confusion, a pathway that nature does not follow, and could not have the intention of following. For since she is determined in all that she does, she never tries to direct her movements in anything besides straight lines.

And thus, if you desire that she would at first direct the light from M to H, she cannot have a mind to set up a ray towards N, since this ray does not have any intention of its own. Rather, she will direct her ray towards R, this line being the straightest, the shortest, and the briefest of all those which could lead to this point, and having arrived there, it will now go from R to H, again the straightest, shortest, and briefest path being a straight line towards H. And thus, if nature acted by your principle, it would have to go directly from M towards H; for on one side she must first direct her ray towards R, and on the other side your principle itself will carry it towards H.

7. And, although you have very clearly demonstrated that according to your assumption the combined time of the two rays MN, NH is shorter than that of any other two lines combined, it is not, however, the brevity of the time that carries these two rays through these two lines.

For would it even be possible for a ray which is already in the air, which already has its direction straight ahead and which does not at all aim elsewhere, then just as soon as it is opposed by water or glass, it has a mind to deflect as it does, for the sole intent of finding a point that will dictate

the briefest composite movement of all those that could have departed from its point of origin? This would be quite a metaphysical reason for a purely material subject.

Should we not rather believe, as I have already said, that since it is the force of the movement and its determination that have directed the ray along the first line that it described, without time having played any role, it is the change that occurs in this force and this determination which cause it to change from the old to the new route, without time having anything to do with it, since time cannot produce anything?

8. Finally, the difference that I find between M. Descartes and yourself is that you do not prove, but instead assume as a principle that light moves more easily through rare bodies than through dense ones: whereas M. Descartes proves, and does not simply make the assumption as you say, that light moves more easily through dense bodies than through rare ones.<sup>6</sup>

For, assuming your principle and also assuming that nature always acts by the shortest or quickest paths, you conclude quite well that light must follow the path that it takes in refraction; and you do this, while M. Descartes, without assuming anything, uses only experience itself to conclude that light moves more easily through dense bodies than through rare ones, and at the same time gives the means of measuring the proportion with which this occurs. And although he judged rightly that the everyday experiment that you have to the contrary could give us cause to be amazed [étonner], he gives the physical reason for it on the twenty-eighth page of his *Dioptrics* that we can have recourse to.<sup>7</sup>

But, if it is true that light moves with more difficulty in rare bodies than in dense ones, as this reasoning put forward by M. Descartes seems to prove, and if it is also true that nature does not always act along the quickest paths, which the example of the ball which passes from air into water justifies us in thinking, then so much for your entire demonstration.<sup>8</sup>

And similarly, since you claim to have posed difficulties for M. Descartes in the past, *viventi atque sentienti*,<sup>9</sup> without either him or his friends ever having satisfied you, could we not also say that he responded to you when

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<sup>6</sup> *This* coming from the man who clearly never threw a rock into a pond to see the path that it takes?! (See section 5, second paragraph.)

<sup>7</sup> Find out what he said on this page!

<sup>8</sup> *adieu toute votre démonstration*

<sup>9</sup> “when he was alive and sentient”

he was alive, and through his friends after his death: *tibi, inquam, viventi, et nisi dici nefas esset, adderem: et non intelligenti?*<sup>10</sup> After all, there are those who are convinced they understand it well.

And finally, since you say that nature seems to have had this deference and complacency for M. Descartes to have given herself up to him and to have revealed her truths to him without having been forced to by a demonstration, can we not say that you have forced Geometry, strict as she is, to provide you with a demonstration by means of this doubly false position?<sup>11</sup>

After this, I leave it to more strict and far-sighted naturalists to judge which of you two has better claimed the cause assigned to refraction. This does not prevent that, in considering things from another standpoint, I might be in agreement with you that nature always acts along the shortest and quickest paths. For, since she only acts with the necessary force, and since she is always determinate in her action, she always does everything that she can; and thus, whatever route she takes, it is always the shortest and the quickest that were possible, considering all of the causes that made her act and which have determined her.

Thus, after having proposed to you what is making me persist in my first impression, I cannot but feel obliged to give in and somehow agree with your position; and far from disputing with you the glory of entering into the society of the proof of such an important truth, I believe I have found a means of bringing the two of you into agreement, by leaving to each of you that which belongs to you.

It seems that, since light is the most noble production of nature, she also allows it to act in the most regular and universal manner, and it seems that she has made things such that in her action, all of the principles that she uses in all the other causes all come together in the action of light.

Thus, since the movements of other bodies depend on the force which moves them and on the determination of this force, light, following these laws, sometimes continues on a straight line and sometimes diverges from it, by approaching or moving away from the perpendicular. But although we also see that nature always acts by the shortest paths, it had to be that light had to accommodate itself to this law.

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<sup>10</sup>“that he responded to you while you were alive, and, if it is no crime to say it, I might add: that you did not understand” This is the “unkind phrase” for which Clerselier is forced to apologize in his next letter to Fermat.

<sup>11</sup>This reference to nature giving up her secrets is from section 6 of Fermat’s January 1, 1662 letter to de la Chambre.

M. Descartes has shown that in refraction light follows the ordinary laws of movement of all bodies, and you, Sir, have shown that even though in refraction light seems to make a detour and seems to forget that it must always act by the shortest paths, it nonetheless observes this law with such great precision that it leaves nothing to be desired.

And thus it may be said that you have worked together with M. Descartes to thereby justify nature and to bring reason to her process: he, by way of reasons that are natural and common to all bodies: and you, Sir, by means of mathematical reasons taken from the purest and finest Geometry.

And likewise, since this geometric proof was the most difficult to find and to fathom, I would very much like you to win over him, and from this moment I sign and subscribe to an eternal peace with you, and I no longer wish to raise this question of the ineffectiveness of your principle or of the difference between your principle and his, when they lead to the same thing and teach us the same truth.

I am, etc.