Conservation of What?

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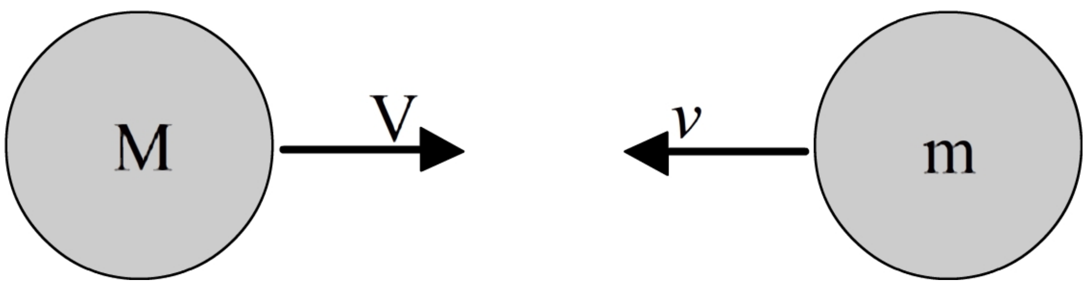
In the first issue of ΔΥΝΑΜΙΣ, I offered a presentation showing that Descartes' conservation of momentum follows easily from Leibniz's twin ideas of *vis viva* and relativity of motion. My demonstration there used specific numbers to make the case, something that I will do here in generality.

Let us take two masses, M,m, moving towards each other with initial speeds (as measured by a stationary observer) V_o,v_o, where v_o may be considered as negative since it is moving backwards:



If this stationary Leibnizian observer were to write down the condition for the masses after the collision, he would say that their velocities V,v are related as:

$$MV_o^2 + mv_o^2 = MV^2 + mv^2$$

This does not give a determinate outcome, since their is a simply extended manifold of possibilities for the two velocities. But let our Leibnizian observer imagine that he were moving to the right with a speed of unity. Would the principle that the full effect is equal to the full cause not still be true? Let us write the conservation of *vis viva* for this second observer. Both velocities will be reduced by unity:

$$M(V_o - 1)^2 + m(v_o - 1)^2 = M(V - 1)^2 + m(v - 1)^2$$

Expanding, we have:

$$MV_o^2 - 2MV_o + M + mv_o^2 - 2mv_o + m = MV^2 - 2MV + M + mv^2 - 2mv + m.$$

Subtracting our original equation:

$$MV_o^2 + mv_o^2 = MV^2 + mv^2,$$

we arrive at

$$-2MV_o + M - 2mv_o + m = -2MV + M - 2mv + m,$$

from which we may subtract $M + m$ from both sides, leaving us with:

$$-2MV_o - 2mv_o = -2MV - 2mv,$$

which, when divided by -2, leaves us:

$$MV_o + mv_o = MV + mv.$$

But, this is simply the conservation of momentum!

Descartes

Now, just to set the record straight, René Descartes did *not* agree with the conservation of momentum. He believed that the *quantity of motion* is maintained, but this is not the same thing: quantity of motion does not depend on direction. Thus, in Descartes' perfect world, a ball bouncing off of the ground begins to move upwards, not because of elastic deformation and reforming, but to maintain the quantity of motion: since the ball can't continue to move downwards, it must move upwards. Descartes tried to maintain a total distinction between speed and direction.

Leibniz's explanation of conservation of momentum in his *Specimen Dynamicum* is a delight, and I think he knew the argument I made using *vis viva* and relativity of motion.